UNIT I
BASIC CIRCUITS ANALYSIS

Ohm's Law – Kirchoffs laws – DC and AC Circuits – Resistors in series and parallel circuits – Meshcurrent and node voltage method of analysis for D.C and A.C. circuits – Phasor Diagram – Power,Power Factor and Energy

1.1.INTRODUCTION:

The interconnection of various electric elements in a prescribed manner comprises as an electric circuit in order to perform a desired function. The electric elements include controlled and uncontrolled source of energy, resistors, capacitors, inductors, etc. Analysis of electric circuits refers to computations required to determinethe unknown quantities such as voltage, current and power associated with one or more elements in the circuit. To contribute to the solution of engineering problems one must acquire the basic knowledge of electric circuit analysis and laws. Many other systems, like mechanical, hydraulic, thermal, magnetic and power system are easy to analyze and model by a circuit. To learn how to analyze the models of these systems, first one needs to learn the techniques of circuit analysis. We shall discuss briefly some of the basic circuit elements and the laws that will help us to develop the background of subject.

1.2. BASIC ELEMENTS & INTRODUCTORY CONCEPTS:

Electrical Network:

A combination of various electric elements (Resistor, Inductor, Capacitor, Voltage source, Current source) connected in any manner what so ever is called an electrical network. We may classify circuit elements in two categories, passiveand active elements.

Passive Element:

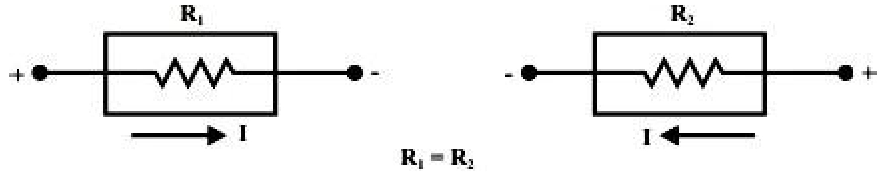
The element which receives energy (or absorbs energy) and then either converts it into heat (R) or stored it in an electric (C) or magnetic (L) field is called passive element.

Active Element:

The elements that supply energy to the circuit is called active element.Examples of active elements include voltage and current sources, generators, and electronic devices that require power supplies. A transistor is an active circuit element, meaning that it can amplify power of a signal. On the other hand, transformer is not an active element because it does not amplify the power level and power remains same both in primary and secondary sides. Transformer is an example of passive element.

Bilateral Element:

Conduction of current in both directions in an element (example: Resistance; Inductance; Capacitance) with same magnitude is termed as bilateral element.



Unilateral Element:

Conduction of current in one direction is termed as unilateral (example: Diode, Transistor) element.

$R_1 \neq R_2$

Meaning of Response:

An application of input signal to the system will produce an output signal, the behavior of output signal with time is known as the response of the system.

Potential Energy Difference:

The voltage or potential energy difference between two points in an electric circuit is the amount of energy required to move a unit charge between the two points.

Ohm's Law: Ohm's law states that the current through a conductor between two points is directly proportional to the potential difference or voltage across the two points, and inversely proportional to the resistance between them. The mathematical equation that describes this relationship is:

$$I = \frac{V}{R}$$

where I is the current through the resistance in units of amperes, V is the potential difference measured across the resistance in units of volts, and R is the resistance of the conductor in units of ohms. More specifically, Ohm's law states that the R in this relation is constant, independent of the current.

1.3. KIRCHOFF'S LAW

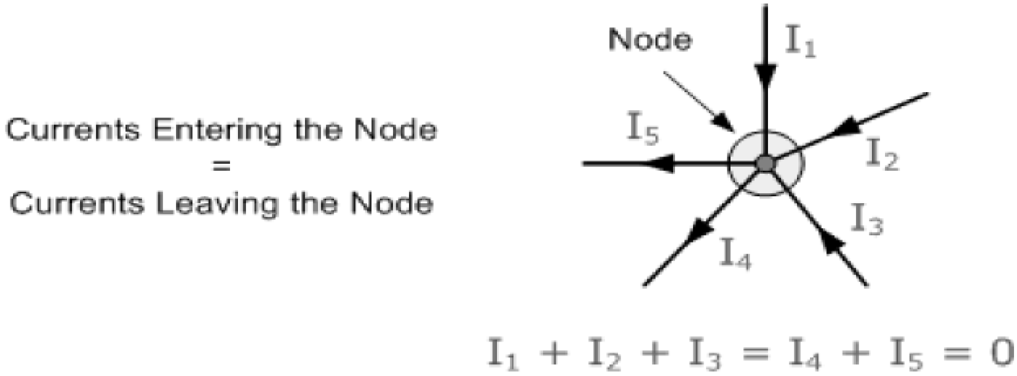
Kirchoff's First Law - The Current Law, (KCL)

"The total current or charge entering a junction or node is exactly equal to the charge leaving the node as it has no other place to go except to leave, as no charge is lost within the node".

In other words the algebraic sum of ALL the currents entering and leaving a node must be equal to zero,

I(exiting) + I(entering) = 0.

This idea by Kirchoff is known as the Conservation of Charge.



Here, the 3 currents entering the node, I1, I2, I3 are all positive in value and the 2 currents leaving the node, I4 and I5 are negative in value.

Then this means we can also rewrite the equation as;

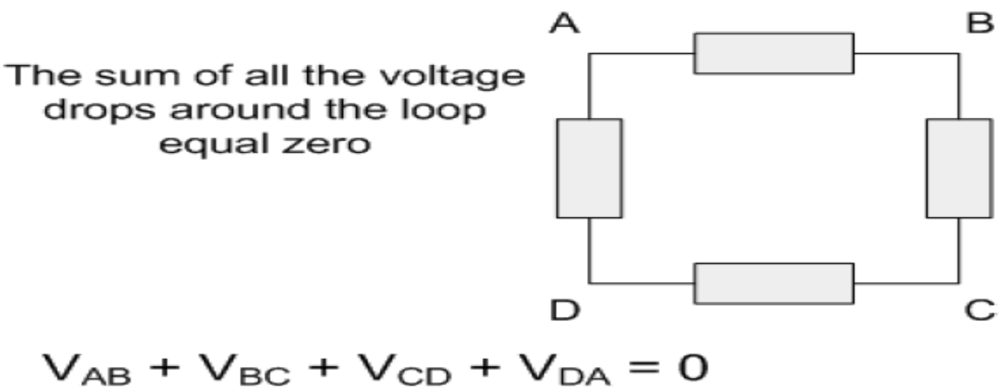
$$I1 + I2 + I3 - I4 - I5 = 0$$

Kirchoff's Second Law - The Voltage Law, (KVL)

"In any closed loop network, the total voltage around the loop is equal to the sum of all the voltage drops within the same loop" which is also equal to zero. In other words the algebraic sum of all voltages within the loop must be equal to zero. This idea by Kirchoff is known as the Conservation of Energy**.**

Starting at any point in the loop continue in the same direction noting the direction of all the voltage drops, either positive or negative, and returning back to the same starting point. It is important to maintain the same direction either clockwise or anti-clockwise or the final voltage sum will not be equal to zero.

We can use Kirchoff's voltage law when analyzing series circuits.



1.4. PROBLEMS AND CALCULATIONS:

Problem 1:

A current of 0.5 A is flowing through the resistance of 10Ω.Find the potential difference between its ends.

Solution:

Current I = 0.5A.

Resistance R = 10Ω

Potential difference V =?

$$V = IR$$
$$= 0.5 \times 10$$
$$= 5V.$$

Problem: 2

A supply voltage of 220V is applied to a 100 Ω resistor. Find the current flowing through it.

Solution:

Voltage V = 220V Resistance R = 100Ω

Current I = V/ R

= 2 2 0 /100

= 2.2 A.

Problem: 3

Calculate the resistance of the conductor if a current of 2A flows through it when the potential difference across its ends is 6V.

Solution:

Current I = 2A.

Potential difference = V = 6.

Resistance R = V/I

= 6 /2

= 3 ohm.

Problem: 4

Calculate the current and resistance of a 100 W, 200V electric bulb.

Solution:

Power, P = 100W

Voltage, V = 200V Power p = VI

Current I = P/V

= 100/200

= 0.5A

Resistance R = V /I

= 200/0.5

= 400W.

Problem: 5

Calculate the power rating of the heater coil when used on 220V supply *taking 5 Amps.*

Solution:

Voltage, V = 220V

Current, I = 5A,

Power, P = VI

= 220 × 5

= 1100W

= 1.1 KW.

Problem: 6

A circuit is made of 0.4 Ω wire, a 150Ω bulb and a 120Ω rheostat *connected in series. Determine the total resistance of the resistance of the circuit.*

Solution:

Resistance of the wire = 0.4Ω Resistance of bulb = 1 5 0 Ω Resistance of rheostat = 120Ω

In series,

Total resistance, R = 0.4 + 150 +120

= 270.4Ω

Problem : 7

Three resistances of values 2Ω,3Ω and 5Ω are connected in series across 20 V,D.C supply .Calculate (a) equivalent resistance of the circuit (b) the total current of the circuit (c) the voltage drop across each resistor and (d) the power dissipated in each resistor.

Solution:

Total resistance R = $R_1 + R_2 + R_3$.

= 2 +3+5 = 10Ω

Voltage = 20V

Total current I = V/R = 20/10 = 2A.

Voltage drop across 2Ω resistor $V_1 = I R_1$

= 2× 2 = 4 volts.

Voltage drop across 3Ω resistor $V_2 = IR_2$

= 2 × 3 = 6 volts.

Voltage drop across 5Ω resistor $V_3 = I R_3$

= 2 ×5 = 10 volts.

Power dissipated in 2Ω resistor is $P_1 = I_2 R_1$

= 22 × 2 = 8 watts.

Power dissipated in 3 resistor is $P_2 = I_2 R_2$.

= 22 × 3 = 12 watts.

Power dissipated in 5 resistor is $P_3 = I_2 R_3$

= 22 × 5 = 20 watts.

Problem: 8

A lamp can work on a 50 volt mains taking 2 amps. What value of the resistance must be connected in series with it so that it can be operated from 200 volt mains giving the same power.

Solution:

Lamp voltage, V = 50V Current, I = 2 amps.

Resistance of the lamp = V/I = 50/25 = 25 Ω

Resistance connected in series with lamp = r.

Supply voltage = 200 volt.

Circuit current I = 2A

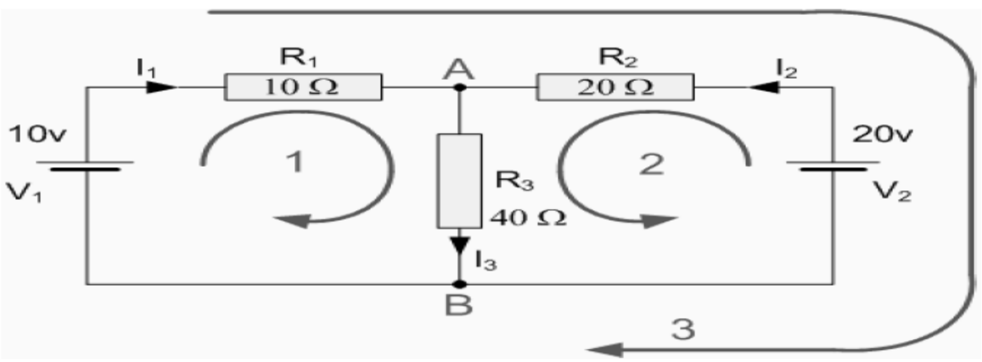
Total resistance R_t = V/I = 200/2 = 100Ω

R_t = R + r 100 = 25 + r

r = 75Ω

Problem: 9

Find the current flowing in the 40Ω Resistor, R3



Solution:

The circuit has 3 branches, 2 nodes (A and B) and 2 independent loops.
Using Kirchoff's Current Law, KCL the equations are given as;
At node A: $I_1 + I_2 = I_3$
At node B: $I_3 = I_1 + I_2$

Using Kirchoff's Voltage Law, KVL the equations are given as;
Loop 1 is given as: $10 = R_1 \text{ x } I_1 + R_3 \text{ x } I_3 = 10I_1 + 40I_3$
Loop 2 is given as: $20 = R_2 \text{ x } I_2 + R_3 \text{ x } I_3 = 20I_2 + 40I_3$
Loop 3 is given as: $10 - 20 = 10I_1 - 20I_2$

As I3 is the sum of $I_1 + I_2$ we can rewrite the equations as;
Eq. No 1: $10 = 10I_1 + 40(I_1 + I_2) = 50I_1 + 40I_2$
Eq. No 2: $20 = 20I_2 + 40(I_1 + I_2) = 40I_1 + 60I_2$

We now have two "Simultaneous Equations" that can be reduced to give us the value of both I_1 and I_2

Substitution of I_1 in terms of I_2 gives us the value of I_1 as -0.143 Amps

Substitution of I_2 in terms of I_1 gives us the value of I_2 as +0.429 Amps

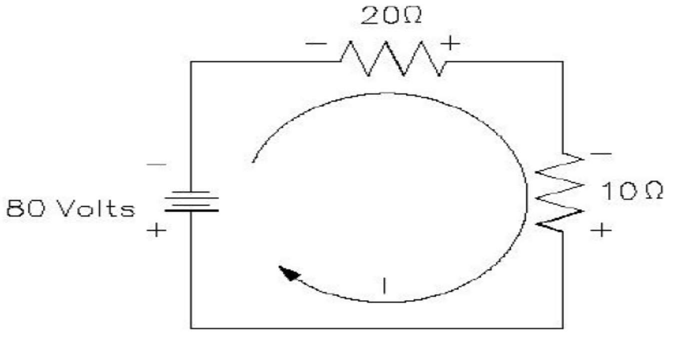
As: $I_3 = I_1 + I_2$

The current flowing in resistor R_3 is given as: -0.143 + 0.429 = 0.286 Amps

and the voltage across the resistor R_3 is given as : 0.286 x 40 = 11.44 volts

Problem: 10

Find the current in a circuit using Kirchhoff's voltage law



Solution:

$$80 = 20(I) + 10(I)$$

$$80 = 30(I)$$

$$I = 80/30 = 2.66 \text{ amperes}$$

1.5. DC CIRCUITS:

- A DC circuit (Direct Current circuit) is an electrical circuit that consists of any combination of constant voltage sources, constant current sources, and resistors. In this case, the circuit voltages and currents are constant, i.e., independent of time. More technically, a DC circuit has no memory. That is, a particular circuit voltage or current does not depend on the past value of any circuit voltage or current. This implies that the system of equations that represent a DC circuit do not involve integrals or derivatives.

- If a capacitor and/or inductor is added to a DC circuit, the resulting circuit is not, strictly speaking, a DC circuit. However, most such circuits have a DC solution. This solution gives the circuit voltages and currents when the circuit is in DC steady state. More technically, such a circuit is represented by a system of differential equations. The solutions to these equations usually contain a time varying or transient part as well as constant or steady state part. It is this steady state part that is the DC solution. There are some circuits that do not have a DC solution. Two simple examples are a constant current source connected to a capacitor and a constant voltage source connected to an inductor.

- In electronics, it is common to refer to a circuit that is powered by a DC voltage source such as a battery or the output of a DC power supply as a DC circuit even though what is meant is that the circuit is DC powered.

1.6. AC CIRCUITS:

Fundamentals of AC:

- An alternating current (AC) is an electrical current, where the magnitude of the current varies in a cyclical form, as opposed to direct current, where the polarity of the current stays constant.
- The usual waveform of an AC circuit is generally that of a sine wave, as these results in the most efficient transmission of energy. However in certain applications different waveforms are used, such as triangular or square waves.
- Used generically, AC refers to the form in which electricity is delivered to businesses and residences. However, audio and radio signals carried on electrical wire are also examples of alternating current. In these applications, an important goal is often the recovery of information encoded (or modulated) onto the AC signal.

1.7. DIFFERENCE BETWEEN AC AND DC:

Current that flows continuously in one direction is called direct current. Alternating current (A.C) is the current that flows in one direction for a brief time then reverses and flows in opposite direction for a similar time. The source for alternating current is called a.c generator or alternator.

Cycle:

- One complete set of positive and negative values of an alternating quantity is called cycle.

Frequency:

- The number of cycles made by an alternating quantity per second is called frequency. The unit of frequency is Hertz (Hz)

Amplitude or Peak value:

- The maximum positive or negative value of an alternating quantity is called amplitude or peak value.

Average value:

- This is the average of instantaneous values of an alternating quantity over one complete cycle of the wave.

Time period:

- The time taken to complete one complete cycle.

Average value derivation:

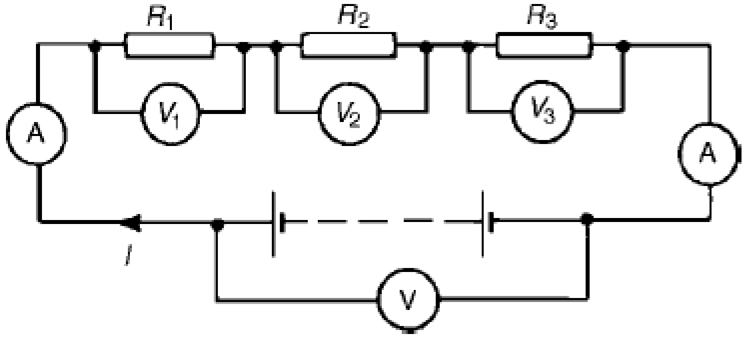
Let i = the instantaneous value of current and $i = I_m \sin \theta$

Where, Im is the maximum value.

Resistors in series and parallel circuits:

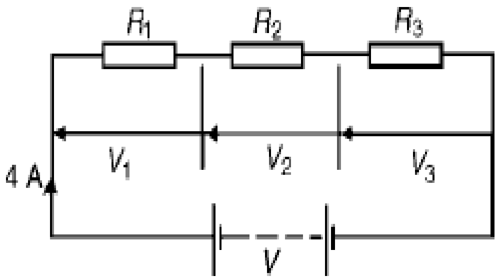
Series circuits:

Figure shows three resistors R1, R2 and R3 connected end to end, i.e. in series, with a battery source of V volts. Since the circuit is closed a current I will flow and the p.d. across each resistor may be determined from the voltmeter readings V1, V2 and V3

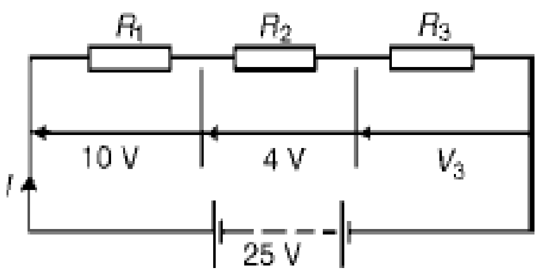


In a series circuit

(a) the current I is the same in all parts of the circuit and hence the same reading is found on each of the two ammeters shown, and
(b) the sum of the voltages V1, V2 and V3 is equal to the total applied voltage, V, i.e.
V = V1 +V2 +V3
From Ohm's law:
V1 = IR1, V2 = IR2, V3 = IR3 and V = IR
where R is the total circuit resistance.
Since V =V1 + V2 + V3
then IR =IR1 + IR2 + IR3
Dividing throughout by I gives
R = R1 + R2 + R3
Thus for a series circuit, the total resistance is obtained by adding together the values of the separate resistances.
Problem 1: For the circuit shown in Figure 5.2, determine (a) the battery voltage V, (b) the total resistance of the circuit, and (c) the values of resistance of resistors R1, R2 and R3, given that the p.d.'s across R1, R2 and R3 are 5V, 2V and 6V respectively.

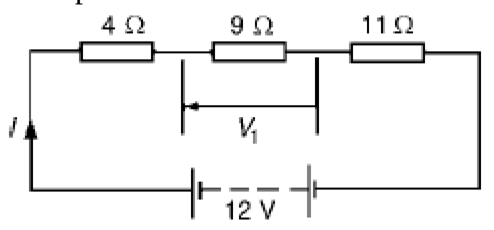


(a) Battery voltage V =V1 + V2 + V3
=5 + 2 + 6=13V
(b) Total circuit resistance R= V/ I
= 13/4=3.25Ω
(c) Resistance R1 = V1/ I
= 5/4
=1.25 Ω
Resistance R2 = V2/ I
= 2/4
=0.5 Ω
Resistance R3 = V3/ I
= 6/4
=1.5 Ω
Problem 2. For the circuit shown in Figure determine the p.d. across resistor *R*3. If the total resistance of the circuit is 100_, determine the current flowing through resistor *R*1. Find also the value of resistor *R*2.



P.d. across R3, V3 =25 − 10 − 4=11V
Current I = V/ R
= 25/100
=0.25A, which is the current flowing in each resistor
Resistance R2 = V2/ I
= 4/0.25
=16 Ω

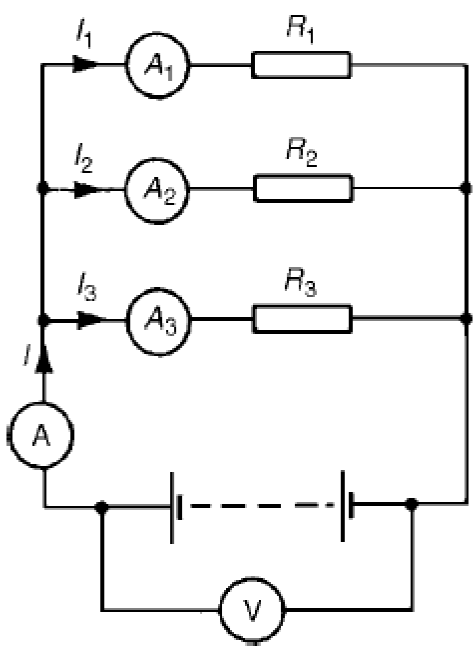
Problem 3: A 12V battery is connected in a circuit having three series-connected resistors having resistances of 4 Ω, 9 Ω and 11 Ω. Determine the current flowing through, and the p.d. across the 9 Ω resistor. Find also the power dissipated in the 11 Ω resistor.



Total resistance R=4 + 9 + 11=24 Ω
Current I = V/ R
= 12/24
=0.5A, which is the current in the 9 Ω resistor.
P.d. across the 9_ resistor, *V*1 = *I* × 9 = 0.5 × 9
= 4.5V
Power dissipated in the 11 Ω resistor, *P* = *I*2*R*=0.52(11)
= 0.25(11)
= 2.75W

1.8. PARALLEL NETWORKS:

Problem 1: Figure shows three resistors, R1, R2 and R3 connected across each other, i.e. in parallel, across a battery source
of V volts.



In a parallel circuit:
(a) the sum of the currents I1, I2 and I3 is equal to the
total circuit current, I, i.e. I =I1 +I2 +I3, and
(b) the source p.d., V volts, is the same across each of the

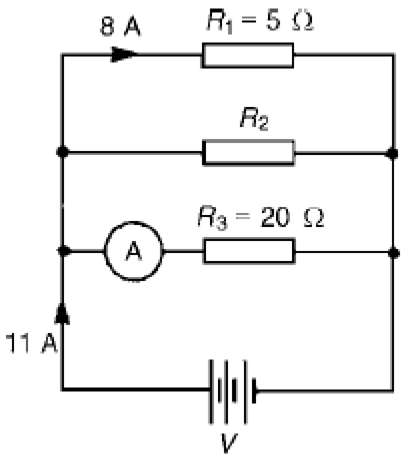
resistors.
From Ohm’s law:
I1 = V/R1
, I2 = V/R2
, I3 = V/R3
and I = V/R
where R is the total circuit resistance.
Since I =I1 + I2 + I3
then
V/R= V/R1+ V/R2+ V/R3
Dividing throughout by V gives:

$$\boxed{\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}$$

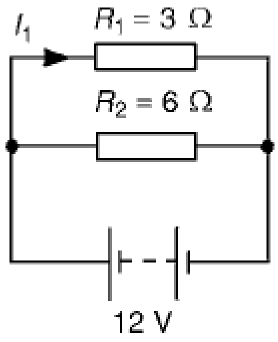
This equation must be used when finding the total resistance R of a parallel circuit. For the special case of two
resistors in parallel

$$\boxed{R=\frac{R_1R_2}{R_1+R_2}}$$

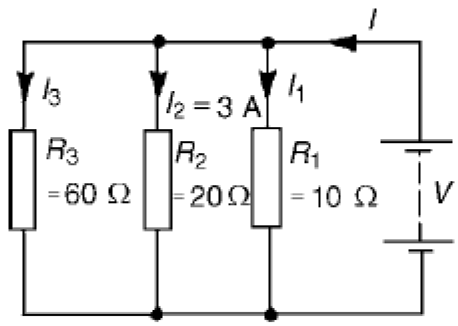
Problem 2: For the circuit shown in Figure , determine (a) the reading on the ammeter, and (b) the value of resistor R2.



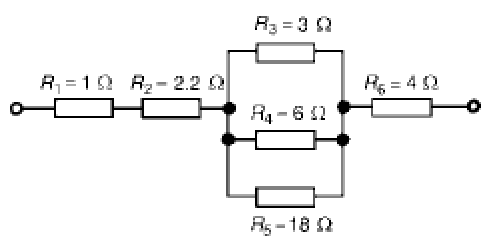
P.d. across R1 is the same as the supply voltage V.
Hence supply voltage, V =8 × 5=40V
(a) Reading on ammeter, I = V
R3= 40/20=2A
(b) Current flowing through R2 =11−8−2=1A
Hence, R2 = V/I2= 40/1=40 Ω



(a) The total circuit resistance R is given by
1/R= 1/R1+ 1/R2= 1/3+ 1/6
1/R= 2 + 1/6= 3/6
Hence, R = 6/3= 2 Ω
(b) Current in the 3 Ω resistance, I1 = V
R1= 12/3= 4A
Problem 3: For the circuit shown in Figure find (a) the value of the supply voltage V and (b) the value of current I.



(a) P.d. across 20 Ω resistor = I2R2 = 3× 20 = 60V, hence supply voltage V =60V since the circuit is connected in parallel.
(b) Current I1 = V/R1= 60/10= 6A; I2 = 3A
I3 = V/R3= 60/60= 1A
Current I =I1+I2+I3 and hence I =6+3+1=10A
Alternatively,
1/R= 1/60+ 1/20+ 1/10= 1 + 3 + 6/60= 10/60
Hence total resistance R= 6010=6 Ω
Current I = V/R= 60/6=10A
Problem 4: Find the equivalent resistance for the circuit shown in Figure



*R*3, *R*4 and*R*5 are connected in parallel and their equivalent resistance *R* is given by:
1/*R*= 1/3+ 1/6+ 1/18=6 + 3 + 1/18= 10/18
Hence *R*= 18/10=1.8 Ω
The circuit is now equivalent to four resistors in series and the equivalent circuit resistance =1+2.2+1.8+4=9 Ω

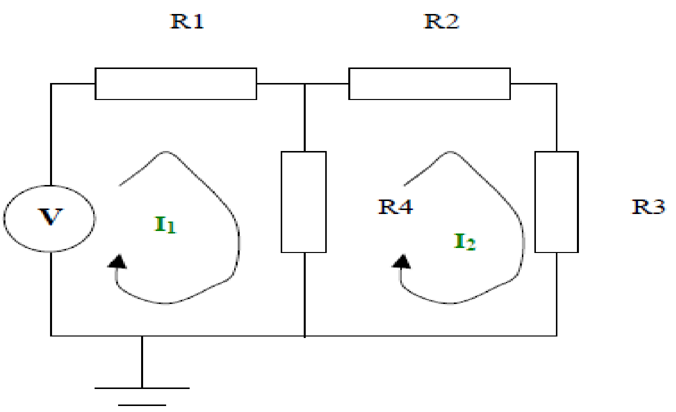
1.9. MESH ANALYSIS:

This is an alternative structured approach to solving the circuit and is based on calculating mesh currents. A similar approach to the node situation is used. A set of equations (based on KVL for each mesh) is formed and the equations are solved for unknown values. As many equations are needed as unknown mesh currents exist.

Step 1: Identify the mesh currents
Step 2: Determine which mesh currents are known
Step 2: Write equation for each mesh using KVL and that includes the mesh currents
Step 3: Solve the equations

Step 1:
The mesh currents are as shown in the diagram on the next page

Step 2:
Neither of the mesh currents is known



Step 3:
KVL can be applied to the left hand side loop. This states the voltages around the loop sum to zero. When writing down the voltages across each resistor Ohm's law is used. The currents used in the equations are the mesh currents.

$I_1R_1 + (I_1 - I_2) R_4 - V = 0$

KVL can be applied to the right hand side loop. This states the voltages around the loop sum to zero. When writing down the voltages across each resistor Ohm's law is used. The currents used in the equations are the mesh currents.

$I_2R_2 + I_2R_3 + (I_2 - I_1) R_4 = 0$

Step 4:
Solving the equations we get

$$I_1 = V\frac{R_2 + R_3 + R_4}{R_1R_2 + R_1R_3 + R_1R_4 + R_2R_4 + R_3R_4}$$

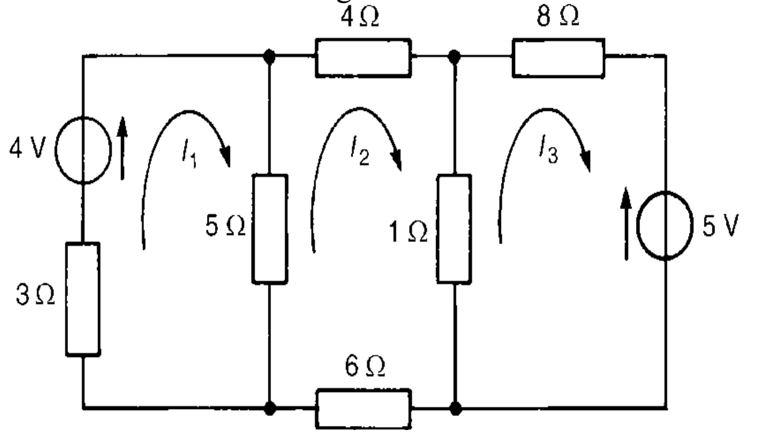
$$I_2 = V\frac{R_4}{R_1R_2 + R_1R_3 + R_1R_4 + R_2R_4 + R_3R_4}$$

The individual branch currents can be obtained from the these mesh currents and the node voltages can also be calculated using this information. For example:

$$V_C = I_2 R_3 = V \frac{R_3 R_4}{R_1 R_2 + R_1 R_3 + R_1 R_4 + R_2 R_4 + R_3 R_4}$$

Problem 1:

Use mesh-current analysis to determine the current flowing in (a) the 5Ω resistance, and (b) the 1Ω resistance of the d.c. circuit shown in Figure.



The mesh currents I_1, I_2 and I_3 are shown in Figure

Using Kirchhoff's voltage law:

For loop 1, $(3 + 5)\ I_1 - I_2 = 4$ ....(1)

For loop 2, $(4 + 1 + 6 + 5)\ I_2 - (5)\ I_1 - (1)\ I_3 = 0$...(2)

For loop 3, $(1 + 8)\ I_3 - (1)\ I_2 = -5$ ....(3)

Thus

$8I_1 - 5I_2 - 4 = 0$

$-5I_1 + 16I_2 - I_3 = 0$

$- I_2 + 9I_3 + 5 = 0$

$$\frac{I_1}{\begin{vmatrix} -5 & 0 & -4 \\ 16 & -1 & 0 \\ -1 & 9 & 5 \end{vmatrix}} = \frac{-I_2}{\begin{vmatrix} 8 & 0 & -4 \\ -5 & -1 & 0 \\ 0 & 9 & 5 \end{vmatrix}} = \frac{I_3}{\begin{vmatrix} 8 & -5 & -4 \\ -5 & 16 & 0 \\ 0 & -1 & 5 \end{vmatrix}}$$

$$= \frac{-1}{\begin{vmatrix} 8 & -5 & 0 \\ -5 & 16 & -1 \\ 0 & -1 & 9 \end{vmatrix}}$$

Using determinants,

$$\frac{I_1}{-5\begin{vmatrix} -1 & 0 \\ 9 & 5 \end{vmatrix} - 4\begin{vmatrix} 16 & -1 \\ -1 & 9 \end{vmatrix}} = \frac{-I_2}{8\begin{vmatrix} -1 & 0 \\ 9 & 5 \end{vmatrix} - 4\begin{vmatrix} -5 & -1 \\ 0 & 9 \end{vmatrix}}$$

$$= \frac{I_3}{-4\begin{vmatrix} -5 & 16 \\ 0 & -1 \end{vmatrix} + 5\begin{vmatrix} 8 & -5 \\ -5 & 16 \end{vmatrix}}$$

$$= \frac{-1}{8\begin{vmatrix} 16 & -1 \\ -1 & 9 \end{vmatrix} + 5\begin{vmatrix} -5 & -1 \\ 0 & 9 \end{vmatrix}}$$

$$\frac{I_1}{-5(-5) - 4(143)} = \frac{-I_2}{8(-5) - 4(-45)}$$

$$= \frac{I_3}{-4(5) + 5(103)}$$

$$= \frac{-1}{8(143) + 5(-45)}$$

$$\frac{I_1}{-547} = \frac{-I_2}{140} = \frac{I_3}{495} = \frac{-1}{919}$$

$$\text{Hence } I_1 = \frac{547}{919} = 0.595 \text{ A},$$

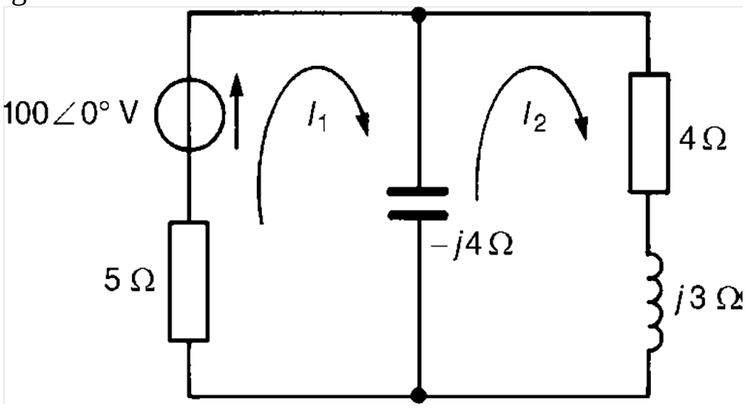
$$I_2 = \frac{140}{919} = 0.152 \text{ A, and}$$

$$I_3 = \frac{-495}{919} = -0.539 \text{ A}$$

(a) Current in the 5 Ω resistance = $I_1 - I_2$
= 0.595 − 0.152
= 0.44A

(b) Current in the 1 Ω resistance = $I_2 - I_3$
= 0.152 − (−0.539)
= 0.69A

Problem 2: For the a.c. network shown in Figure determine, using mesh-current analysis, (a) the mesh currents *I1* and *I2* (b) the current flowing in the capacitor, and (c) the active power delivered by the 100∠0◦V voltage source.



(a) For the first loop

$(5-j4)\ I_1 - (-j4I_2) = 100\angle 0^{\circ}$ .... (1)

For the second loop

$(4+j3-j4)I_2 - (-j4I_1) = 0$ .... (2)

Rewriting equations (1) and (2) gives:

$(5 - j4)I_1 + j4I_2 - 100 = 0$

$j4I_1 + (4 - j)\ I2 + 0 = 0$

Thus, using determinants,

$$\frac{I_1}{\begin{vmatrix} j4 & -100 \\ (4-j) & 0 \end{vmatrix}} = \frac{-I_2}{\begin{vmatrix} (5-j4) & -100 \\ j4 & 0 \end{vmatrix}} = \frac{1}{\begin{vmatrix} (5-j4) & j4 \\ j4 & (4-j) \end{vmatrix}}$$

$$\frac{I_1}{(400 - j100)} = \frac{-I_2}{j400} = \frac{1}{(32 - j21)}$$

Hence $I_1 = \dfrac{(400 - j100)}{(32 - j21)} = \dfrac{412.31\angle -14.04^{\circ}}{38.28\angle -33.27^{\circ}}$

$= 10.77\angle 19.23^{\circ}$ A $=$ **$10.8\angle -19.2^{\circ}$ A,** correct to one decimal place

$I_2 = \dfrac{400\angle -90^{\circ}}{38.28\angle -33.27^{\circ}} = 10.45\angle -56.73^{\circ}$ A

$=$ **$10.5\angle -56.7^{\circ}$ A,**

(b) Current flowing

$= I_1 - I_2$

$= 10.77\angle 19.$

$= 4.44 + j12$

i.e. **the current**

(c) Source power P

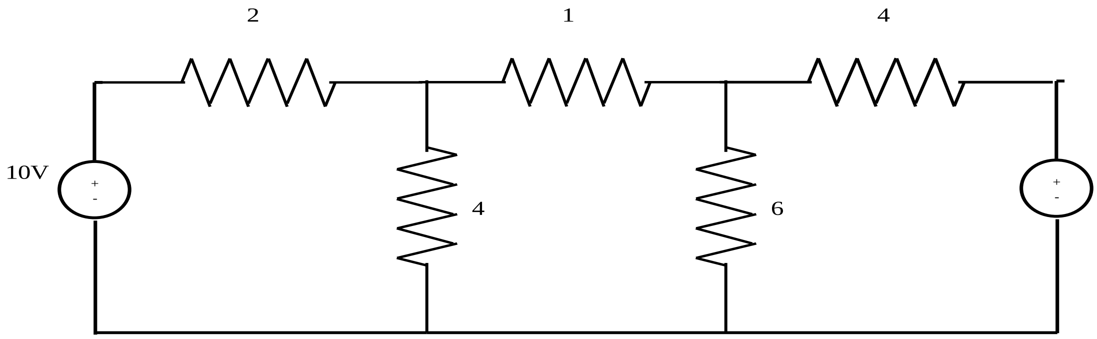
(Check: power i

anc

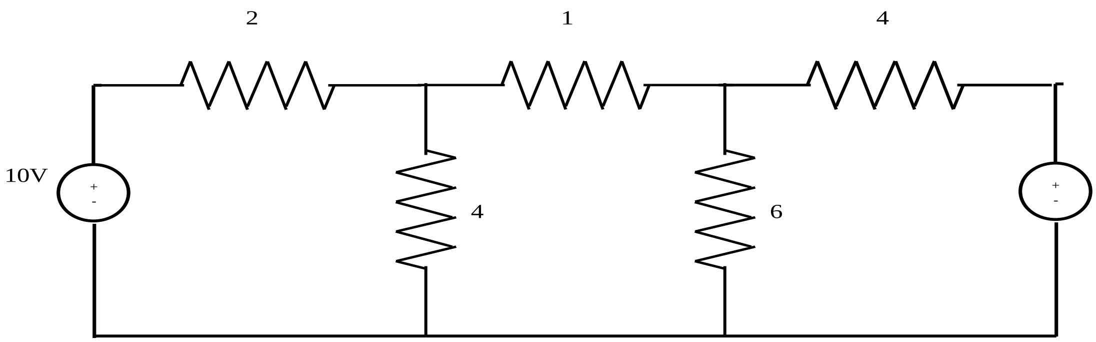
Thus total power dissipated = 579.97 + 436.81

= 1016.8W = 1020W

Problem 3: Calculate current through 6Ω resistance using loop analysis.(AU-JUNE-12)



Solution:



Case(1): Consider loop ABGH ; Apply KVL .

$10 = 2I1 + 4(I_1 - I_2)$

$10 = 6I_1 - 4I_2$ -------------------- (1)

Consider loop BCFG

$I_2 + 6(I_2 + I_3) + 4(I_2 - I_1) = 0$

$1_1 I_2 + 6I_3 - 4I_1 = 0$ ------------- (2)

Consider loop CDEF

$20 = 4I_3 + 6(I_2 = I_3)$

$20 = 10I_3 + 6I_2$ ----------------- (3)

$$D = \begin{vmatrix} 6 & -4 & 0 \\ -4 & 11 & 6 \\ 0 & 6 & 10 \end{vmatrix}$$

$$= \begin{vmatrix} 10 \\ 0 \\ 20 \end{vmatrix}$$

$D = [\ 6\ (110 - 36) + 4(-40)] = 284.$

$$D_1 = \begin{vmatrix} 10 & -4 & 0 \\ 0 & 11 & 6 \\ 20 & 6 & 10 \end{vmatrix}$$

$D_1 = 10[110-36+ (-120)]$

$= 260$

$$D_2 = \begin{vmatrix} 6 & 10 & 0 \\ -4 & 0 & 6 \\ 0 & 20 & 10 \end{vmatrix}$$

$D_2 = 6(-120)-10(-40) = -320$

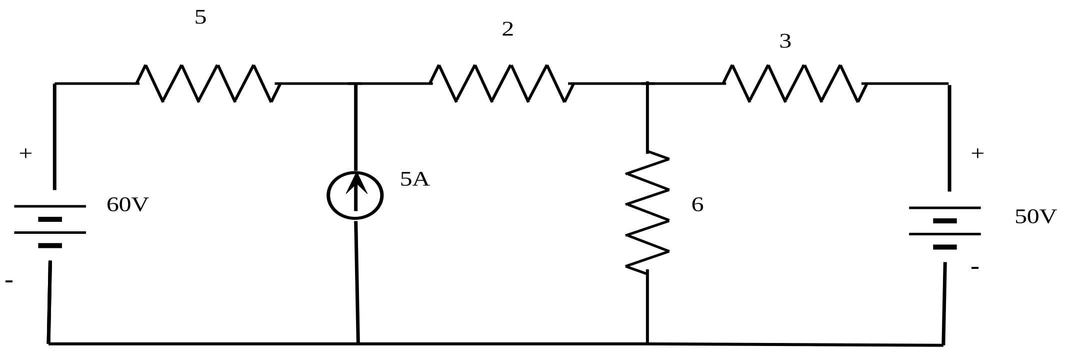
$$D_3 = \begin{vmatrix} 6 & -4 & 10 \\ -4 & 11 & 0 \\ 0 & 6 & 20 \end{vmatrix}$$

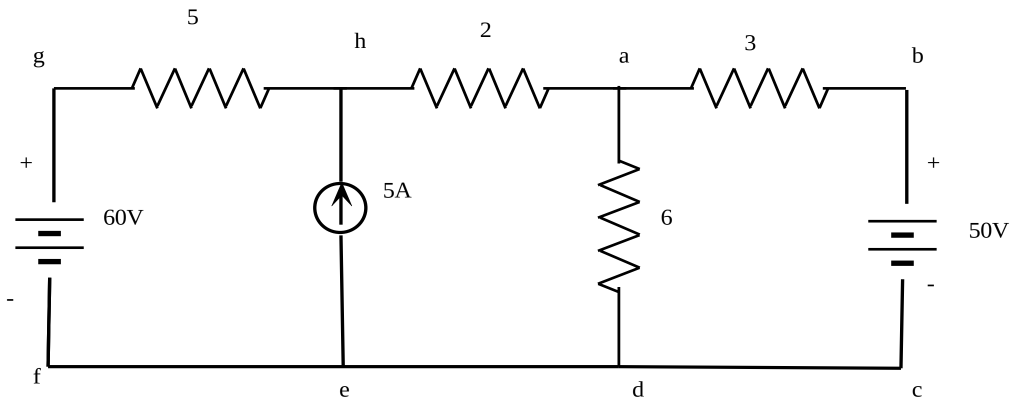
$D_3 = 6(220) +4(-80) +10(-24)$
$D_3 = 760$

$I_1 = D_1/D = 260/284 = 0.915A$
$I_2 = D_2/D = -320/284 = -1.1267A$
$I_3 = D_3/D = 760/284 = 2.676A$

Current through 6Ω resistance $= I_2+I_3$
$= -1.1267+2.676 = 1.55A$

Problem 4: Find the current through branch a-b using mesh analysis. (JUN-09)





Solution:

Consider loops

Loop HADE → $5I_1+2I_2+6(I_2-I_3) = 60$
$5I_1+8I_2-6I_3 = 60$ ------------ (1)

Loop ABCDA→ $3I_3+6(I_3-I_2) = -50$
$3I_3+6I_3-6I_2 = -50$
$9I_3-6I_2 = -50$---------- (2)

$I_2-I_1 = 5A$ ------------------------- (3)

From (1), (2) & (3).

$$D = \begin{vmatrix} -1 & 1 & 0 \\ 5 & 8 & -6 \\ 0 & -6 & 9 \end{vmatrix}$$

$= -1(72-36)-1(45)$

$D = -81.$

$$D_3 = \begin{vmatrix} -1 & 1 & 5 \\ 5 & 8 & 60 \\ 0 & -6 & -50 \end{vmatrix}$$

$= -1(-400+360)-(-250)+5(-30)$

$= 40+250-150$

$D_3 = 140.$

$I_3 = D_3/D = 140/-81 = -1.7283$

The current through branch ab is 1.7283A which is flowing from b to a.

1.10. NODAL ANALYSIS:

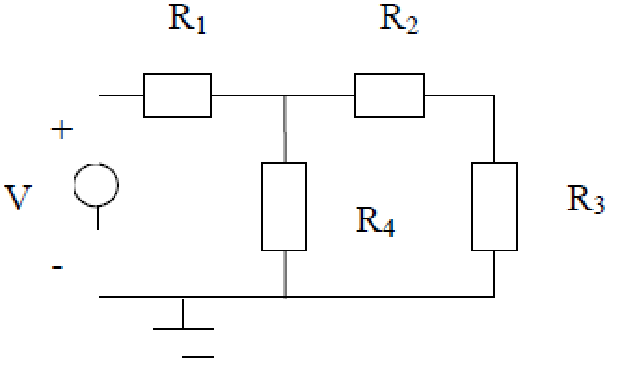
Nodal analysis involves looking at a circuit and determining all the node voltages in the circuit. The voltage at any given node of a circuit is the voltage drop between that node and a reference node (usually ground). Once the node voltages are known any of the currents flowing in the circuit can be determined. The node method offers an organized way of achieving this.

Approach:

Firstly all the nodes in the circuited are counted and identified. Secondly nodes at which the voltage is already known are listed. A set of equations based on the node voltages are formed and these equations are solved for unknown quantities. The set of equations are formed using KCL at each node. The set of simultaneous equations that is produced is then solved. Branch currents can then be found once the node voltages are known. This can be reduced to a series of steps:

Step 1: Identify the nodes
Step 2: Choose a reference node
Step 3: Identify which node voltages are known if any
Step 4: Identify the BRANCH currents
Step 5: Use KCL to write an equation for each unknown node voltage
Step 6: Solve the equations

This is best illustrated with an example. Find all currents and voltages in the following circuit using the node method. (In this particular case it can be solved in other ways as well)



Step 1:

There are four nodes in the circuit. A, B, C and D

Step 2:

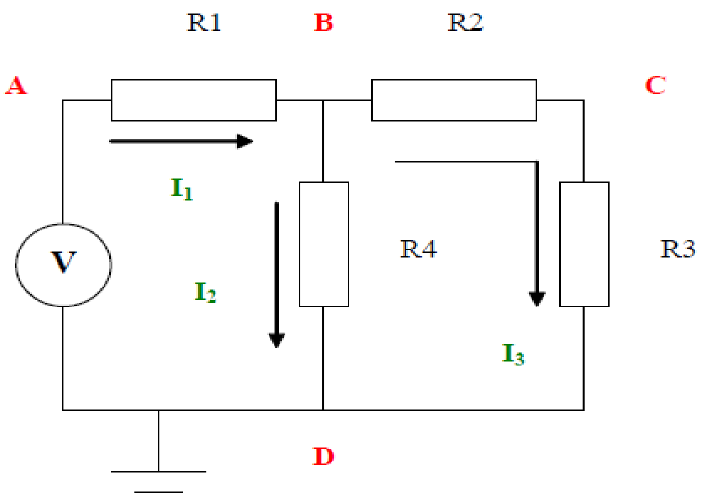
Ground, node D is the reference node.

Step 3:

Node voltage B and C are unknown. Voltage at A is V and at D is 0

Step 4:

The currents are as shown. There are 3 different currents



Step 5:
I need to create two equations so I apply KCL at node B and node C
The statement of KCL for node B is as follows:

$$\frac{V - V_B}{R_1} + \frac{V_C - V_B}{R_2} + \frac{-V_B}{R_4} = 0$$

The statement of KCL for node C is as follows:

$$\frac{V_C - V_B}{R_2} + \frac{-V_B}{R_3} = 0$$

Step 6:
We now have two equations to solve for the two unknowns VB and VC. Solving the above two equations we get:

$$V_C = V \frac{R_3 R_4}{R_1 R_2 + R_1 R_3 + R_1 R_4 + R_2 R_4 + R_3 R_4}$$

$$V_B = V \frac{R_4 (R_2 + R_3)}{R_1 R_2 + R_1 R_3 + R_1 R_4 + R_2 R_4 + R_3 R_4}$$

Further Calculations
The node voltages are know all known. From these we can get the branch currents by a simple application of Ohm's Law:

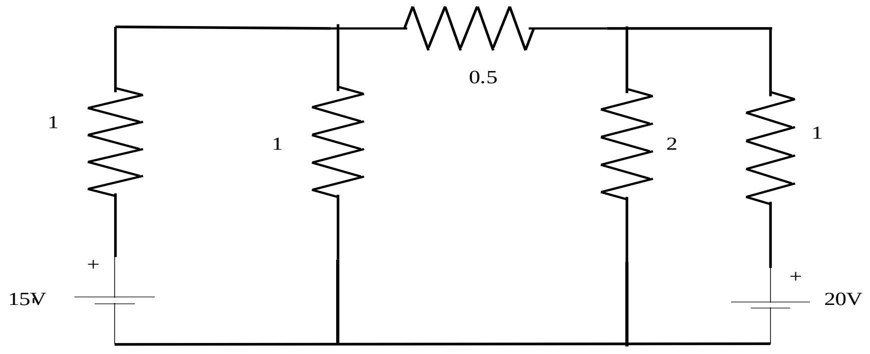
$$I_1 = (V - V_B) / R_1$$

$$I_2 = (V_B - V_C) / R_2$$

$$I_3 = (V_C) / R_3$$

$$I_4 = (V_B) / R_4$$

Problem 1: Find the current through each resistor of the circuit shown in fig, using nodal analysis



Solution:

At node1,

$-I_1-I_2-I_3 = 0$
$-[V_1-15/1]-[V_1/1][V_1-V_2/0.5] = 0$
$-V_1+15-V_1-2V_1+2V_2 = 0$
$4V_1-2V_2 = 15$ --------------------- (1)

At node2,

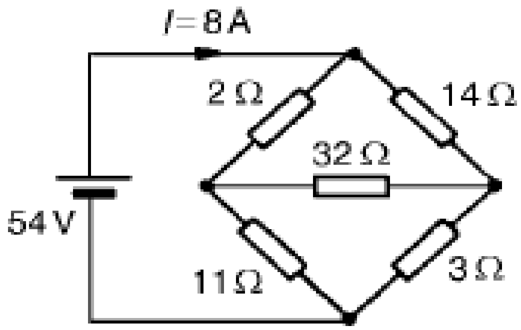
I_3-I4-I5 = 0
$V_1-V_2/0.5 – V_2/2 – V_2-20/1 = 0$
$2V_1-2V_2-0.5V_2-V_2 + 20 = 0$
$2V_1-3.5V_2 = -20$ ------------------ (2)

Multiplying (2) by 2 & subtracting from (1)

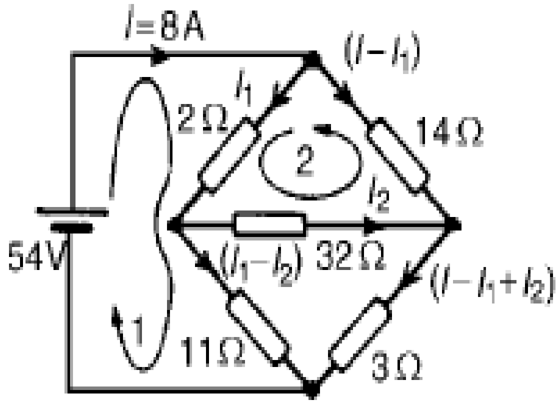
$5V_2 = 55$
$V_2 = 11V$
$V = 9.25V$

$I_1 = V_1-5/1 = 9.25-15 = -5.75A = 5.75$
$I_2 = V_1/1 = 9.25A$
$I_3 = V_1-V_2/0.5 = -3.5A = 3.5A \leftarrow$
I4 = $V_2/2 = 5.5A$
I5 = V_2-20/I = 11-20/1 = -9A=9A.

Problem 2: For the bridge network shown in Figure determine the currents in each of the resistors. (DEC-07)



Let the current in the 2 resistor be I_1, and then by Kirchhoff's current law, the current in the 1_4 resistor is $(I - I_1)$. Let the current in the 32 resistor be I_2 as shown in Figure Then the current in the 1_1 resistor is $(I_1 - I_2)$ and that in the 3 resistor is $(I - I_1 + I_2)$. Applying Kirchhoff's voltage law to loop 1 and moving in a clockwise direction as shown in Figure gives:



$54 = 2I_1 + 11(I_1 \_-I_2)$

i.e. $13I_1 - 11I_2 = 54$

Applying Kirchhoff's voltage law to loop 2 and moving in an anticlockwise direction as shown in Figure gives:

$$0 = 2I_1 + 32I_2 - 14(I - I_1)$$

However I = 8 A

Hence $0 = 2I_1 + 32I_2 – 14(8 -\_I_1)$

i.e. $16I_1 + 32I_2 = 11_2$

Equations (1) and (2) are simultaneous equations with two unknowns, I_1 and I_2.

16 * (1) gives: $208I_1 \_-176I_2 = 864$

13 * (2) gives: $208I_1 + 416I_2 = 1456$

(4) - (3) gives: $592I_2 = 592$, $I_2 = 1$ A

Substituting for I2 in (1) gives:

$13I_1 - 1_1 = 54$

$I_1 = 65/1_3 = 5$ A

Hence,

the current flowing in the 2 resistor $= I_1 = 5$ A

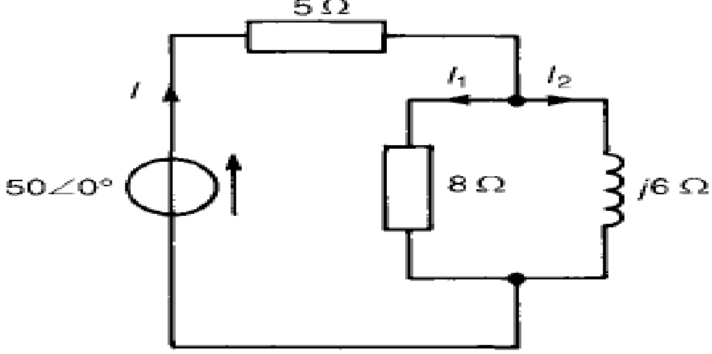
the current flowing in the 14 resistor $= I - I_1 = 8 - 5 = 3$ A

the current flowing in the 32 resistor $= I_2 = 1$ A

the current flowing in the 11 resistor $= I_1 - I_2 = 5 - 1 = 4$ A and

the current flowing in the 3 resistor $= I - I_1 + I_2 = 8 - 5 + 1 = 4$ A

Problem 2: Determine the values of currents I, I1 and I2 shown in the network of Figure



Total circuit impedance,

Z_T = 5 + (8)(j6)/8 + j6

= 5 + (j48)(8 - j6)/8^2 + 6^2

= 5 + (j384 + 288)/100

= (7.88 + j3.84) or 8.776 25.98° A

Current I = V/Z_T

= 50∟ 0°/8.77∟ 25.98°

= 5.7066 −25.98° A

Current I1 = I (j6/8 + j6)

= (5.702∟5.98°) (6∟ 90°)/10∟ 36.87°

= 3.426∟27.15° A

Current I2 = I (8/ (8 + j6)

= (5.70∟-25.98°)*8∟ 0°/10∟ 36.87°

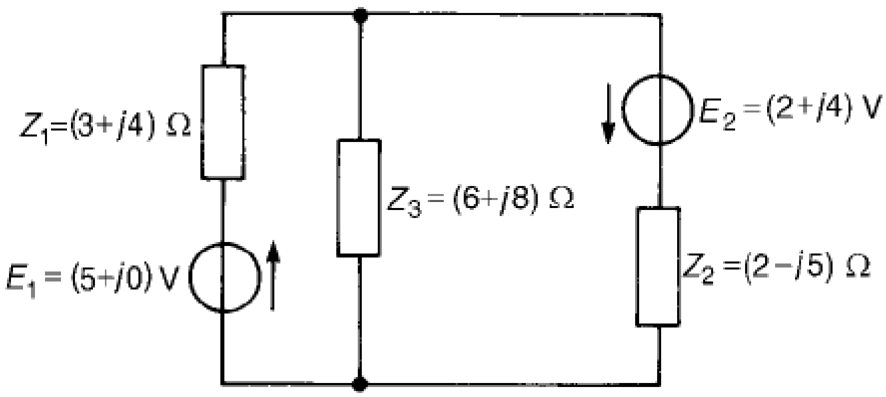
= 4.5666 −62.85° A

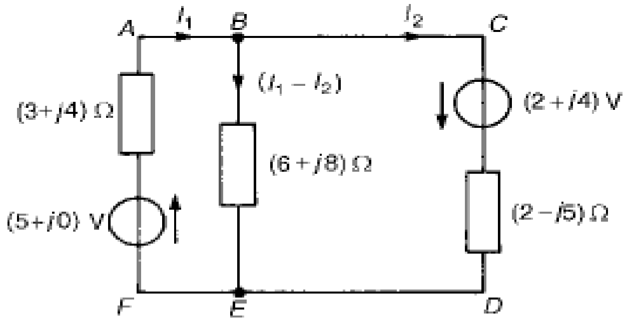
[Note: I = I1 + I2 = 3.42 ∟27.15° + 4.56∟-62.85°

= 3.043 + j1.561 + 2.081 - j4.058

= 5.124 - j2.497 A = 5.706 -25.98° A

Problem 4: For the a.c. network shown in Figure, determine the current flowing in each branch using Kirchhoff's laws.





$$\text{from which, } \boldsymbol{I_1} = \frac{20 + j55}{64 + j27} = \frac{58.52\angle 70.02^\circ}{69.46\angle 22.87^\circ} = \mathbf{0.842\angle 47.15^\circ\ A}$$

$$= (0.573 + j0.617)\ \text{A}$$

$$= \mathbf{(0.57 + j0.62)\ A,}\ \text{correct to two decimal places.}$$

From equation (1), $5 = (9 + j12)(0.573 + j0.617) - (6 + j8)I_2$

$$5 = (-2.247 + j12.429) - (6 + j8)I_2$$

$$\text{from which, } \boldsymbol{I_2} = \frac{-2.247 + j12.429 - 5}{6 + j8}$$

$$= \frac{14.39\angle 120.25^\circ}{10\angle 53.13^\circ}$$

$$= \mathbf{1.439\angle 67.12^\circ A} = (0.559 + j1.326)\ \text{A}$$

$$= \mathbf{(0.56 + j1.33)A,}\ \text{correct to two decimal places.}$$

The current in the $(6 + j8)\Omega$ impedance,

$$\boldsymbol{I_1 - I_2} = (0.573 + j0.617) - (0.559 + j1.326)$$

$$= \mathbf{(0.014 - j0.709)A}\ \text{or}\ \mathbf{0.709\angle -88.87^\circ\ A}$$

An alternative method of solving equations (1) and (2) is shown below, using determinants.

$$(9 + j12)I_1 - (6 + j8)I_2 - 5 = 0 \qquad (1)$$

$$-(6 + j8)I_1 + (8 + j3)I_2 - (2 + j4) = 0 \qquad (2)$$

$$\text{Thus}\quad \frac{I_1}{\begin{vmatrix} -(6+j8) & -5 \\ (8+j3) & -(2+j4) \end{vmatrix}} = \frac{-I_2}{\begin{vmatrix} (9+j12) & -5 \\ -(6+j8) & -(2+j4) \end{vmatrix}}$$

$$= \frac{1}{\begin{vmatrix} (9+j12) & -(6+j8) \\ -(6+j8) & (8+j3) \end{vmatrix}}$$

$$\frac{I_1}{(-20+j40)+(40+j15)} = \frac{-I_2}{(30-j60)-(30+j40)}$$

$$= \frac{1}{(36+j123)-(-28+j96)}$$

$$\frac{I_1}{20+j55} = \frac{-I_2}{-j100} = \frac{1}{64+j27}$$

$$\text{Hence}\quad I_1 = \frac{20+j55}{64+j27} = \frac{58.52\angle 70.02^\circ}{69.46\angle 22.87^\circ}$$

$$= \mathbf{0.842\angle 47.15^\circ\ A}$$

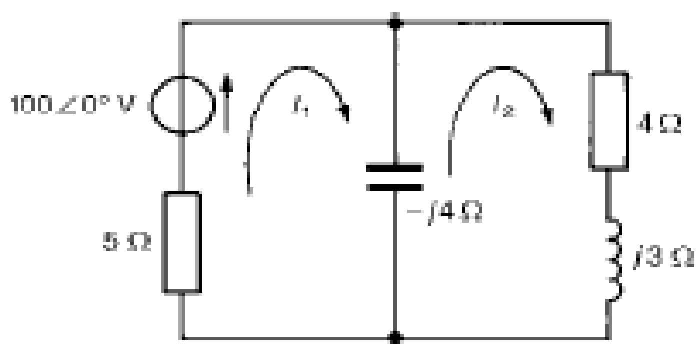
$$\text{and}\qquad I_2 = \frac{100\angle 90^\circ}{69.46\angle 22.87^\circ} = \mathbf{1.440\angle 67.13^\circ\ A}$$

The current flowing in the $(6+j8)\ \Omega$ impedance is given by:

$$I_1 - I_2 = 0.842\angle 47.15^\circ - 1.440\angle 67.13^\circ A$$

$$= \mathbf{(0.013 - j0.709)\ A\ \text{or}\ 0.709\angle -88.95^\circ\ A}$$

Problem 5: For the a.c. network shown in Figure determine, using mesh-current analysis, (a) the mesh currents I_1 and I_2 (b) the current flowing in the capacitor, and (c) the active power delivered by the 1006 0° V voltage source.



(a) For the first loop $(5 - j4)I_1 - (-j4I_2) = 100\angle 0°$ (1)

For the second loop $(4 + j3 - j4)I_2 - (-j4I_1) = 0$ (2)

Rewriting equations (1) and (2) gives:

$$(5 - j4)I_1 + j4I_2 - 100 = 0 \qquad (1')$$

$$j4I_1 + (4 - j)I_2 + 0 = 0 \qquad (2')$$

Thus, using determinants,

$$\frac{I_1}{\begin{vmatrix} j4 & -100 \\ (4-j) & 0 \end{vmatrix}} = \frac{-I_2}{\begin{vmatrix} (5-j4) & -100 \\ j4 & 0 \end{vmatrix}} = \frac{1}{\begin{vmatrix} (5-j4) & j4 \\ j4 & (4-j) \end{vmatrix}}$$

$$\frac{I_1}{(400 - j100)} = \frac{-I_2}{j400} = \frac{1}{(32 - j21)}$$

Hence $$I_1 = \frac{(400 - j100)}{(32 - j21)} = \frac{412.31\angle -14.04°}{38.28\angle -33.27°}$$

$$= 10.77\angle 19.23° \text{ A} = \mathbf{10.8\angle -19.2° \text{ A}},$$

correct to one decimal place

$$I_2 = \frac{400\angle -90°}{38.28\angle -33.27°} = 10.45\angle -56.73° \text{ A}$$

$$= \mathbf{10.5\angle -56.7° \text{ A}},$$

correct to one decimal place

(b) Current flowing in capacitor $= I_1 - I_2$

$$= 10.77\angle 19.23° - 10.45\angle -56.73°$$

$$= 4.44 + j12.28 = 13.1\angle 70.12° \text{ A},$$

i.e., **the current in the capacitor is 13.1 A**

(c) Source power $P = VI\cos\phi = (100)(10.77)\cos 19.23^\circ$

$= \mathbf{1016.9\ W} = 1020$ W,

correct to three significant figures.

(Check: power in 5 Ω resistor $= I_1^2(5) = (10.77)^2(5) = 579.97$ W

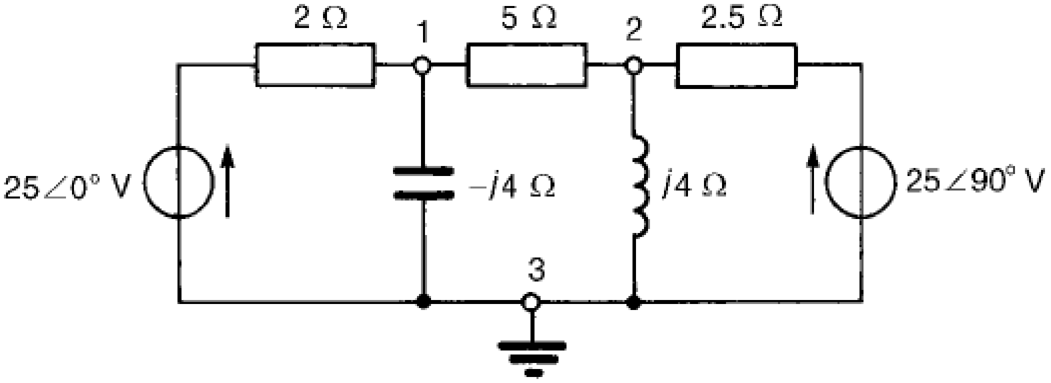
and power in 4 Ω resistor $= I_2^2(4) = (10.45)^2(4) = 436.81$ W

Thus total power dissipated $= 579.97 + 436.81$

$= 1016.8$ W $= 1020$ W, correct

to three significant figures.)

Problem 6: In the network of Figure use nodal analysis to determine (a) the voltage at nodes 1 and 2, (b) the current in the j4 Ω inductance, (c) the current in the 5 Ω resistance, and (d) the magnitude of the active power dissipated in the 2.5 Ω resistance.(AU DEC-10)



(a) At node 1, $\dfrac{V_1 - 25\angle 0^\circ}{2} + \dfrac{V_1}{-j4} + \dfrac{V_1 - V_2}{5} = 0$

Rearranging gives:

$$\left(\frac{1}{2} + \frac{1}{-j4} + \frac{1}{5}\right)V_1 - \left(\frac{1}{5}\right)V_2 - \frac{25\angle 0^\circ}{2} = 0$$

i.e., $$(0.7 + j0.25)V_1 - 0.2V_2 - 12.5 = 0 \qquad (1)$$

At node 2, $\dfrac{V_2 - 25\angle 90^\circ}{2.5} + \dfrac{V_2}{j4} + \dfrac{V_2 - V_1}{5} = 0$

Rearranging gives:

$$-\left(\frac{1}{5}\right)V_1 + \left(\frac{1}{2.5} + \frac{1}{j4} + \frac{1}{5}\right)V_2 - \frac{25\angle 90^\circ}{2.5} = 0$$

i.e., $$-0.2V_1 + (0.6 - j0.25)V_2 - j10 = 0 \qquad (2)$$

Thus two simultaneous equations have been formed with two unknowns, V_1 and V_2. Using determinants, if

$$(0.7 + j0.25)V_1 - 0.2V_2 - 12.5 = 0 \qquad (1)$$

and $$-0.2V_1 + (0.6 - j0.25)V_2 - j10 = 0 \qquad (2)$$

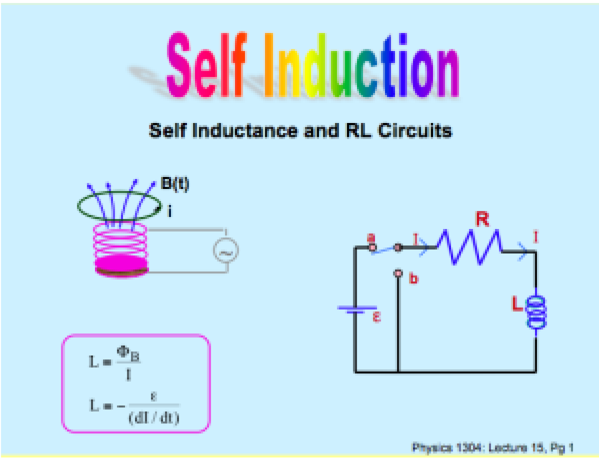
then $$\frac{V_1}{\begin{vmatrix} -0.2 & -12.5 \\ (0.6 - j0.25) & -j10 \end{vmatrix}} = \frac{-V_2}{\begin{vmatrix} (0.7 + j0.25) & -12.5 \\ -0.2 & -j10 \end{vmatrix}}$$

$$= \frac{1}{\begin{vmatrix} (0.7 + j0.25) & -0.2 \\ -0.2 & (0.6 - j0.25) \end{vmatrix}}$$

1.1.Self inductance

When current changes in a circuit, the magnetic flux linking the same circuit changes and e.m.f is induced in the circuit. This is due to the self inductance, denoted by L.

$$V = L\frac{di}{dt}$$



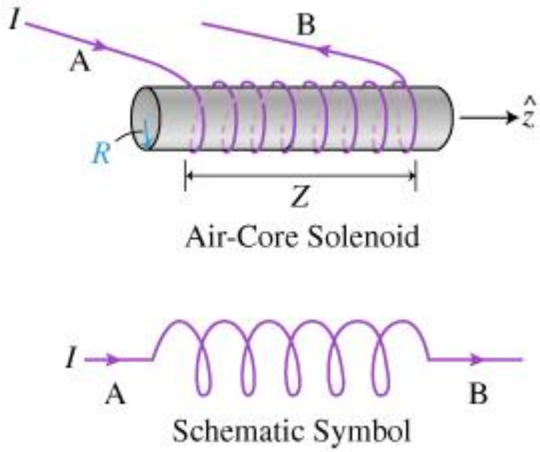


FIG.1

1.2.Mutual Inductance

The total magnetic flux linkage in a linear inductor made of a coil is proportional to the current

passing through it; that is,

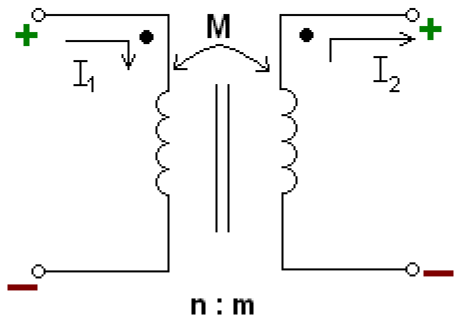


Fig. 2

$\lambda = Li$

. By Faraday's law, the voltage across the inductor is equal to the time derivative of the total influx linkage; given by,

$$L\frac{di}{dt} = N\frac{d\phi}{dt}$$

1.3. **Coupling Coefficient**

A coil containing N turns with magnetic flux Ø_ linking each turn has total magnetic flux linkage λ=NØ

. By Faraday's law, the induced emf (voltage) in the coil is

$$e = -\left(\frac{d\lambda}{dt}\right) = -N\left(\frac{d\phi}{dt}\right)$$

. A negative sign is frequently included in this equation to signal that the voltage polarity is established according to Lenz's law. By definition of self-inductance this voltage is also given by Ldi=dtÞ; hence,

The unit of flux(Ø) being the weber, where 1 Wb = 1 V s, it follows from the above relation that 1 H = 1 Wb/A. Throughout this book it has been assumed that Ø and i are proportional to each other, making

L = *(NØ) /I* = constant

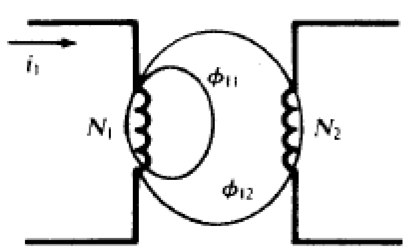


Fig.3

In Fig.3 , the total flux resulting from current i1 through the turns N1 consists of leakage flux, Ø11, and coupling or linking flux, Ø12. The induced emf in the coupled coil is given by *$N_2(dØ_{12}/dt)$*. This same voltage can be written using the mutual inductance M:

$$M = N_1 \frac{d\phi_{21}}{di_2}$$

$$e = M\frac{di_1}{dt} = N_2\frac{d\phi_{12}}{dt}$$

or

$$M = N_2 \frac{d\phi_{12}}{di_1}$$

Also, as the coupling is bilateral,

$$M = N_1 \frac{d\phi_{21}}{di_2}$$

$$M^2 = \left(N_2\frac{d\phi_{12}}{dt}\right)\left(N_1\frac{d\phi_{21}}{di_2}\right)$$

$$= \left(N_2\frac{(k\phi_1)}{di_1}\right)\left(N_1\frac{(k\phi_2)}{di_2}\right)$$

$$= k^2\left(N_1\frac{d\phi_1}{di_1}\right)\left(N_2\frac{(d\phi_2)}{di_2}\right)$$

$$= k^2 L_1 L_2$$

Hence,mutual inductance , M is given by

$$M = k\sqrt{L_1 L_2}$$

And the mutual reactance X_M is given by

$$X_M = k\sqrt{X_1}X_2$$

The coupling coefficient, k, is defined as the ratio of linking flux to total flux:

$$k = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}$$

1.4.Series connection of coupled circuit (lecture 2)

When inductors are connected together in series so that the magnetic field of one links with the other, the effect of mutual inductance either increases or decreases the total inductance depending upon the amount of magnetic coupling. The effect of this mutual inductance depends upon the distance apart of the coils and their orientation to each other.

Mutually connected inductors in series can be classed as either “Aiding” or “Opposing” the total inductance. If the magnetic flux produced by the current flows through the coils in the same direction then the coils are said to be **Cumulatively Coupled**. If the current flows through the coils in opposite directions then the coils are said to be **Differentially Coupled** as shown below.

1.4.1.Cumulatively Coupled Series Inductors

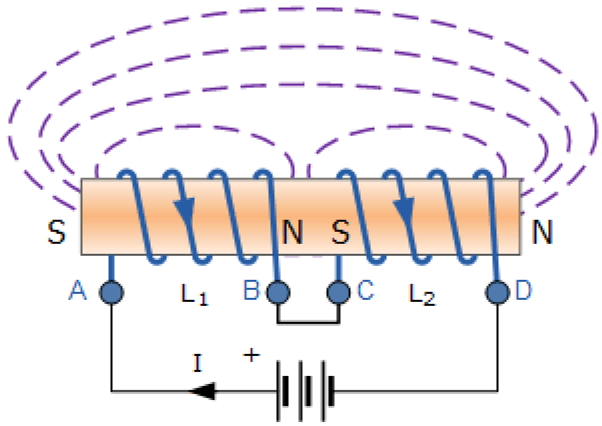


Fig.4

While the current flowing between points A and D through the two cumulatively coupled coils is in the same direction, the equation above for the voltage drops across each of the coils needs to be modified to take into account the interaction between the two coils due to the effect of mutual inductance. The self inductance of each individual coil, L_1 and L_2 respectively will be the same as before but with the addition of M denoting the mutual inductance.

Then the total emf induced into the cumulatively coupled coils is given as:

$$L_T \frac{di}{dt} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + 2\left(M \frac{di}{dt}\right)$$

Where: 2M represents the influence of coil L_1 on L_2 and likewise coil L_2 on L_1.

By dividing through the above equation by di/dt we can reduce it to give a final expression for calculating the total inductance of a circuit when the inductors are cumulatively connected and this is given as:

$$L_{total} = L_1 + L_2 + 2M$$

If one of the coils is reversed so that the same current flows through each coil but in opposite directions, the mutual inductance, M that exists between the two coils will have a cancelling effect on each coil as shown below.

1.4.2.Differentially Coupled Series Inductors

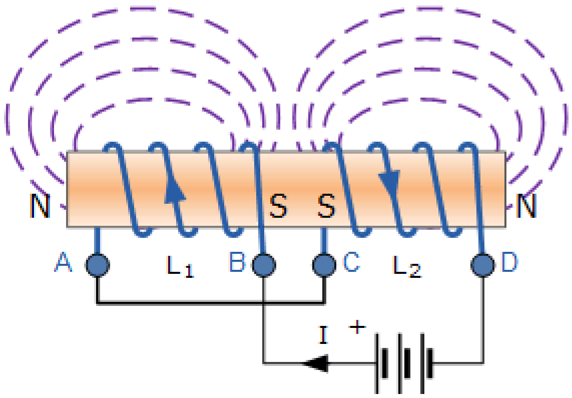


Fig.5

The emf that is induced into coil 1 by the effect of the mutual inductance of coil 2 is in opposition to the self-induced emf in coil 1 as now the same current passes through each coil in opposite directions. To take account of this cancelling effect a minus sign is used with M when the magnetic field of the two coils are differentially connected giving us the final equation for calculating the total inductance of a circuit when the inductors are differentially connected as:

$$L_{total} = L_1 + L_2 - 2M$$

Then the final equation for inductively coupled inductors in series is given as:

$$L_T = L_1 + L_2 \pm 2M$$

Inductors in Series Example No2

Two inductors of 10mH respectively are connected together in a series combination so that their magnetic fields aid each other giving cumulative coupling. Their mutual inductance is given as 5mH. Calculate the total inductance of the series combination.

$$L_T = L_1 + L_2 + 2M$$
$$L_T = 10mH + 10mH + 2(5mH)$$
$$L_T = 30mH$$

Inductors in Series Example No3

Two coils connected in series have a self-inductance of 20mH and 60mH respectively. The total inductance of the combination was found to be 100mH. Determine the amount of mutual inductance that exists between the two coils assuming that they are aiding each other.

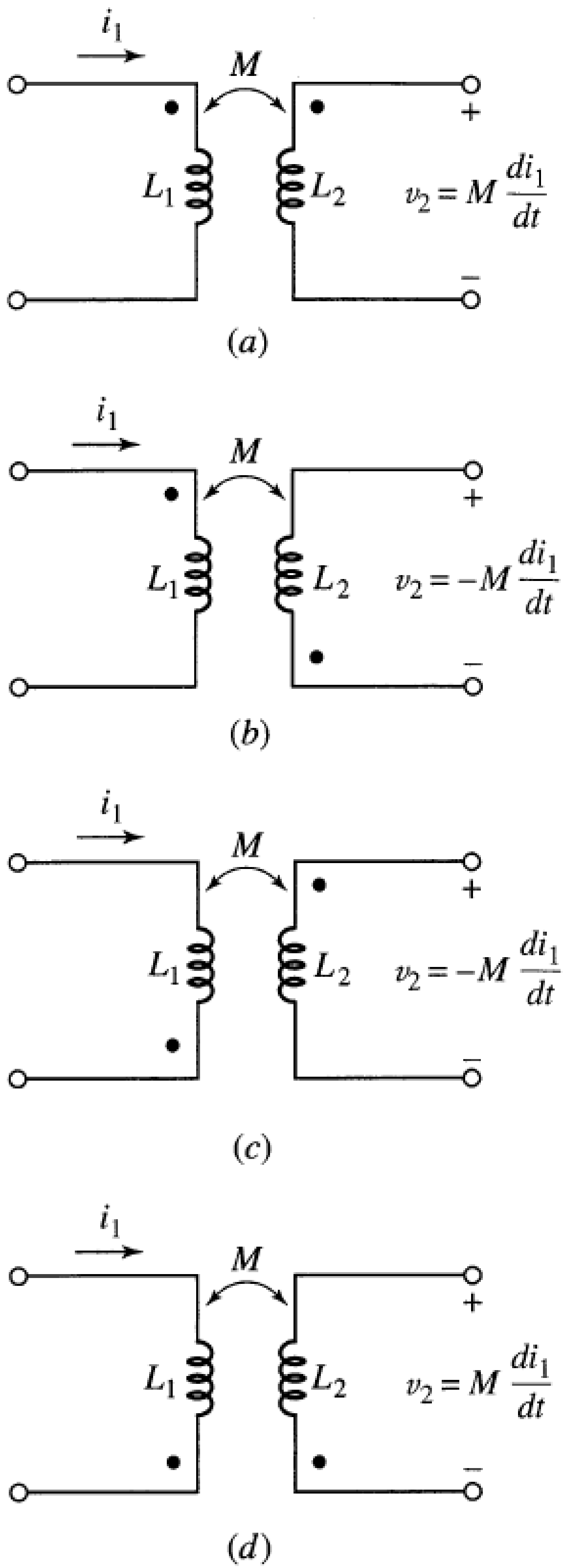
$$L_T = L_1 + L_2 \pm 2M$$

$$100 = 20 + 60 + 2M$$

$$2M = 100 - 20 - 60$$

$$\therefore M = \frac{20}{2} = 10mH$$

1.4.DOT RULE (lecture 3)



i_1
M
L_1
L_2
+
−
$v_2 = M \frac{di_1}{dt}$
(a)
i_1
M
L_1
L_2
+
−
$v_2 = -M \frac{di_1}{dt}$
(b)
i_1
M
L_1
L_2
+
−
$v_2 = -M \frac{di_1}{dt}$
(c)
i_1
M
L_1
L_2
+
−
$v_2 = M \frac{di_1}{dt}$
(d)

Fig.6

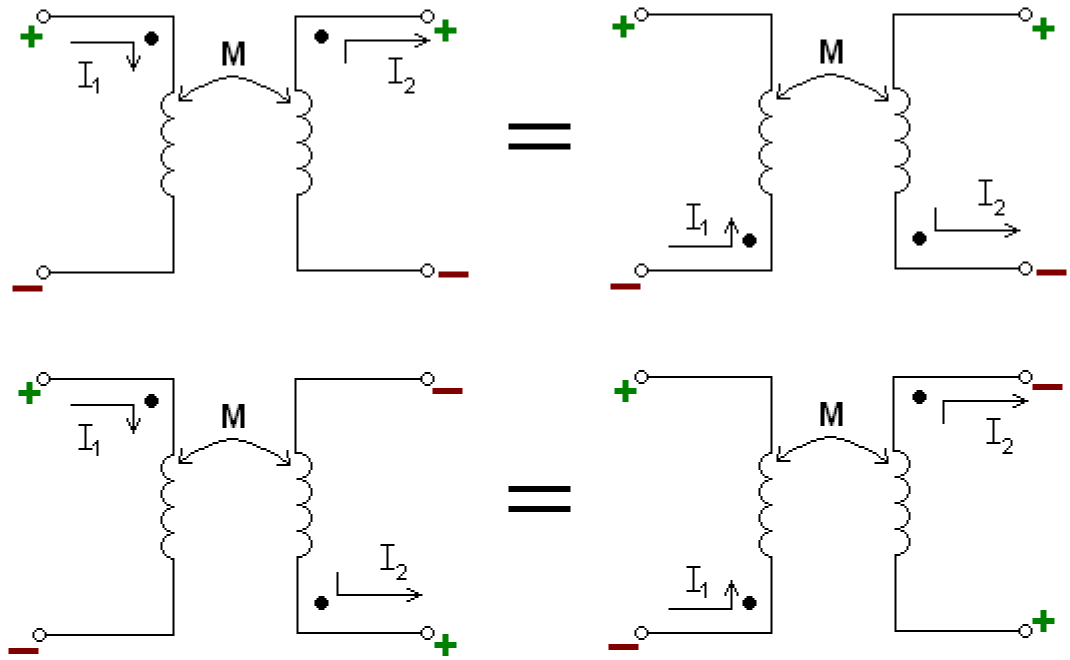


Fig.7

The sign on a voltage of mutual inductance can be determined if the winding sense is shown on the circuit diagram, as in Fig. To simplify the problem of obtaining the correct sign, the coils are marked with dots at the terminals which are instantaneously of the same polarity. To assign the dots to a pair of coupled coils, select a current direction in one coil and place a dot at the terminal where this current enters the winding. Determine the corresponding flux by application of the right-hand rule [see Fig. 14-7(a)]. The flux of the other winding, according to Lenz's law, opposes the first flux. Use the right-hand rule to find the natural current direction corresponding to this second flux [see Fig. 14-7(b)]. Now place a dot at the terminal of the second winding where the natural current leaves the winding. This terminal is positive simultaneously with the terminal of the first coil where the initial current entered. With the instantaneous polarity of the coupled coils given by the dots, the pictorial representation of the core with its winding sense is no longer needed, and the coupled coils may be illustrated as in Fig. 14-7(c). The following dot rule may now be used:

(1) when the assumed currents both enter or both leave a pair of coupled coils by the dotted terminals, the signs on the M-terms will be the same as the signs on the L-terms; but

(2) if one current enters by a dotted terminal while the other leaves by a dotted terminal, the signs

on the M-terms will be opposite to the signs on the L-terms

1.5.CONDUCTIVELY COUPLED EQUIVALENT CIRCUITS

From the mesh current equations written for magnetically coupled coils, a conductively coupled
equivalent circuit can be constructed. Consider the sinusoidal steady-state circuit of Fig. 14-9(a), with
the mesh currents as shown. The corresponding equations in matrix form are

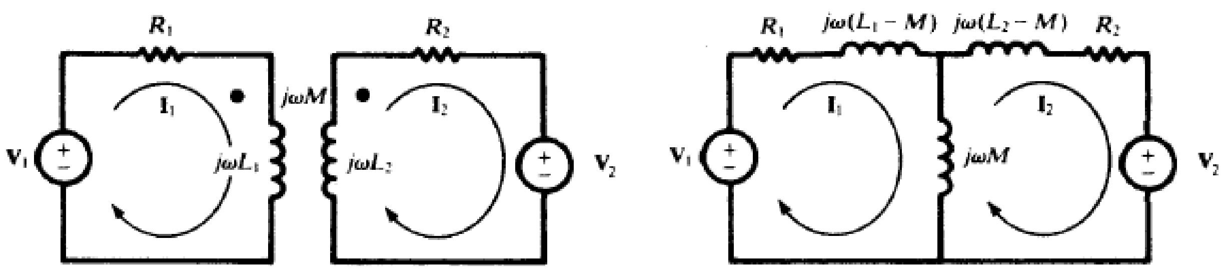


Fig.8

$$\begin{bmatrix} R_1 + j\omega L_1 & -j\omega M \\ -j\omega M & R_2 + j\omega M_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

1.6.Single tuned coupled circuit

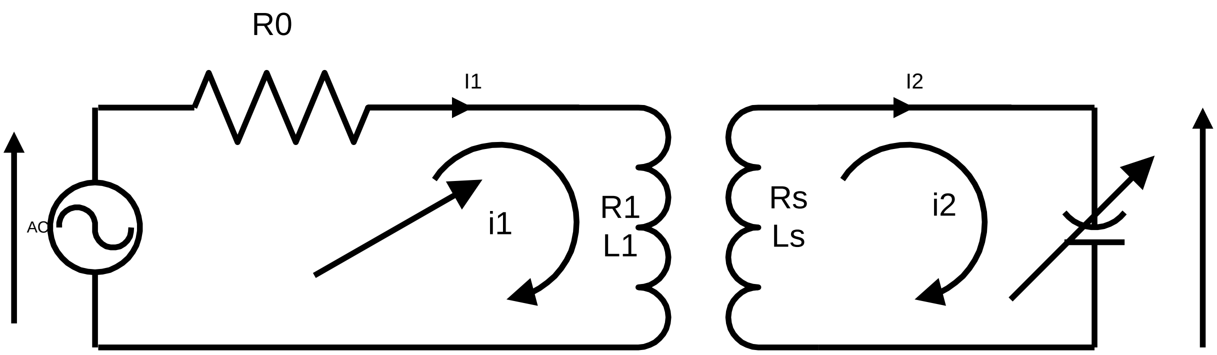


Fig.9

$$Z_{11}I_1 - Z_{12}I_2 = E$$
$$-Z_{21}I_1 + Z_{22}I_2 = 0$$

$$I_2 = \frac{\begin{bmatrix} Z_{11} & E \\ -Z_{21} & 0 \end{bmatrix}}{\begin{bmatrix} Z_{11} & -Z_{12} \\ -Z_{21} & Z_{22} \end{bmatrix}} = \frac{EZ_{21}}{Z_{11}Z_{22} - Z_{12}Z_{21}}$$

$$I_2 = \frac{E(j\omega M)}{R_1(R_2 + jX_2) + \omega^2 M^2}$$

For

$$\omega L \ll R$$

$$I_2 = \frac{E(j\omega M)}{R_1\left[R_2 + j\left(\omega Ls - \frac{1}{\omega Cs}\right)\right] + \omega^2 M^2}$$

$$V_O = \frac{EM}{C_s\left\{R_1\left[R_2 + j\left(\omega Ls - \frac{1}{\omega Cs}\right)\right] + \omega^2 M^2\right\}}$$

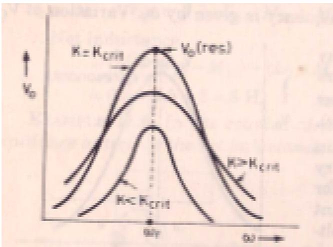


FIG. variation of V_0 with ω for different values of K

$$A = \frac{V_O}{E} = \frac{\cancel{E}M}{\cancel{E}C_s\left\{R_1\left[R_2 + j\left(\omega Ls - \frac{1}{\omega Cs}\right)\right] + \omega^2 M^2\right\}}$$

At resonance

$$I_{2r} = \frac{EM}{R_1 R_2 + \omega_r{}^2 M^2}$$

$$V_{2r} = \left(\frac{E(\omega_r M)}{R_1 R_2 + \omega_r{}^2 M^2}\right) \times \frac{1}{C_s}$$

$$A_r = \left(\frac{M}{R_1 R_2 + \omega_r{}^2 M^2}\right) \times \frac{1}{C_s}$$

$$M_{opt} = \frac{\sqrt{R_1 R_1}}{\omega_r}$$

1.7.Double tuned coupled circuit.(lecture-4)

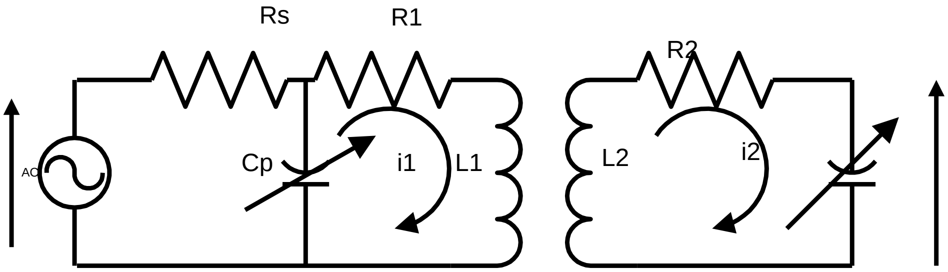


Fig.10

In this circuit the tuning capacitors are placed both in primary as well as secondary side.
From the figure the eq. impedance on primary side is, Z_{11},
Given by

$$Z_{11} = R_0 + R_p + j\left(\omega_p L_p - \frac{1}{\omega C_p}\right)$$

$$= R_1 + jX_1$$

$$Z_{22} = R_s + j\left(\omega L_s - \frac{1}{\omega C_s}\right)$$

$$= R_2 + jX_2$$

$$Z_{12} = Z_{21} = j\omega M$$

Applying loop analysis

$$I_2 = \frac{E_1 Z_{12}}{Z_{11}Z_{12} - Z_{12}^2}$$

$$V_0 = \frac{I_2}{j\omega C_s} = \frac{E_1 M}{C_s\left(R_1 + jX_1\right)\left(R_2 + jX_2\right) + \omega^2 M^2}$$

$$= \frac{E_1 M}{C_s\left(\left(R_1 + jX_1\right)\left(R_2 + jX_2\right) + \omega^2 M^2\right)}$$

$$A = \frac{V_0}{E_1} = \frac{E_1 M}{C_s\left(\left(R_1 + jX_1\right)\left(R_2 + jX_2\right) + \omega^2 M^2\right)} \div E_1$$

$$= \frac{\cancel{E_1} M}{C_s\left(\left(R_1 + jX_1\right)\left(R_2 + jX_2\right) + \omega^2 M^2\right)} \times \frac{1}{\cancel{E_1}}$$

$$\boxed{= \frac{M}{C_s\left(\left(R_1 + jX_1\right)\left(R_2 + jX_2\right) + \omega^2 M^2\right)}}$$

At resonance

$$\boxed{I_{2_{res}} = \frac{E_1 \omega_r M}{R_1 R_2 + \omega_r^2 M^2}}$$

$$\boxed{V_{o_{res}} = \frac{E_1 \cancel{\omega_r} M}{R_1 R_2 + \omega_r^2 M^2} \times \frac{1}{\cancel{\omega_r} C_s}}$$

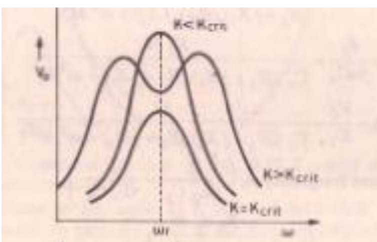


FIG. variation of V_0 with ω for different values of K

Or,

$$= \frac{E_1 M / C_s}{R_1 R_2 + \omega_r^2 M^2}$$

$$A_{res} = \frac{V_{o_{res}}}{E_1} = \frac{E_1 M / C_s}{R_1 R_2 + \omega_r^2 M^2} \div E_1$$

$$= \frac{\cancel{E_1} M}{C_s\left(R_1 R_2 + \omega_r^2 M^2\right)} \times \frac{1}{\cancel{E_1}}$$

$$\boxed{= \frac{M}{C_s\left(R_1 R_2 + \omega_r^2 M^2\right)}}$$

For maximum out put voltage at resonance,

The denominator should be minimum
As

$$\frac{R_1 R_2}{M} = \omega_r^2 M$$

$$M_{opt} = \frac{\sqrt{R_1 R_2}}{\omega_r} = K_{crit}\sqrt{L_p L_s}$$

This value will give the optimum M
M_{opt}

2.Two-Port Networks

2.1.Terminals and Ports (lecture-5)

In a two-terminal network, the terminal voltage is related to the terminal current by the impedance
Z=*V* / *I*.

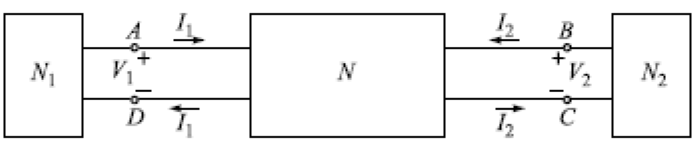


Fig.1

In a four-terminal network, if each terminal pair (or port) is connected separately to another circuit as in Fig. , the four variables i1, i2, v1, and v2 are related by two equations called the terminal characteristics. These two equations, plus the terminal characteristics of the connected circuits, provide the necessary and sufficient number of equations to solve for the four variables.

2.2.***Z*-PARAMETERS (open circuit parameters)**

The terminal characteristics of a two-port network, having linear elements and dependent sources,
may be written in the s-domain as

$$V_1=Z_{11} I_1+Z_{12} I_2 \qquad (1)$$

$$V_2=Z_{21}I_1+Z_{22} I_2 \qquad (2)$$

$$Z_{11} = V_1/ I_1 \qquad \text{(for } I_2=0\text{)}$$

$Z_{21} = V_2/ I_1$ (for I_2 =0)

$Z_{12=}$ V_1 /I_2 (for I_1=0)

$Z_{22} = V_2 /I_2$ (for I_1=0)

2.3.Y-PARAMETERS(short circuit parameters)

The terminal characteristics may also be written as , where I1 and I2 are expressed in terms of V1 and V2.

I1 = Y11V1 + Y12V2 (3)
I2 = Y21V1 + Y22V2 (4)

this yields

Y_{11} =I_1 /V_1 forV_2=0
Y_{11} =I_1/V_2 For V_1=0
Y_{21}=I_2/V_2 for V_2=0
Y_{22} =I_2/V_{22} forV_1=0

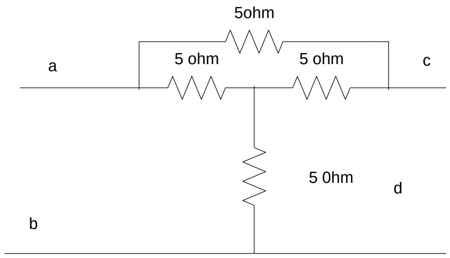
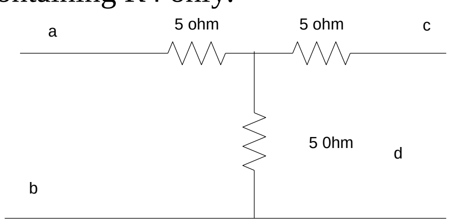


Fig.2
Solution:

We can make two separate neyworks one a T netwotk comprising R1,R2,R3 and anetwork containing R4 only.



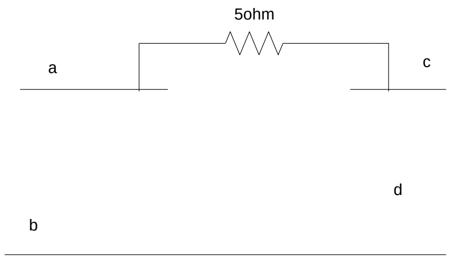
$$Y_{11} = \frac{1}{7.5} mho$$

$$Y_{12} = -\frac{1}{15} mho$$

$$Y_{21} = \frac{1}{15} mho$$

$$Y_{22} = \frac{1}{7.5} mho$$

Similarly for the next network



$$Y_{11} = \frac{1}{5} mho$$

$$Y_{21} = -\frac{1}{5} mho$$

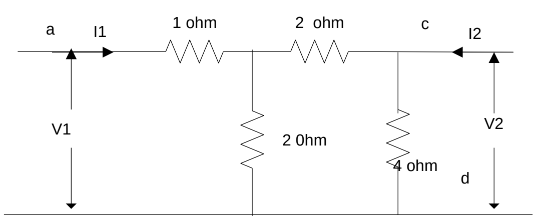
$$Y_{12} = -\frac{1}{5} mho$$

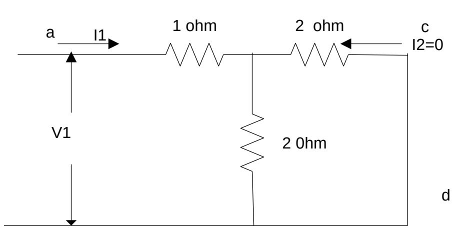
$$Y_{22} = \frac{1}{5} mho$$

$$[Y_{net}] = [Y_T] + [Y_B]$$

$$Y_{net} = \begin{bmatrix} 1/3 & -4/15 \\ -4/15 & 1/3 \end{bmatrix}$$ (ans)

Example.Find the Y parameters for the given circuit



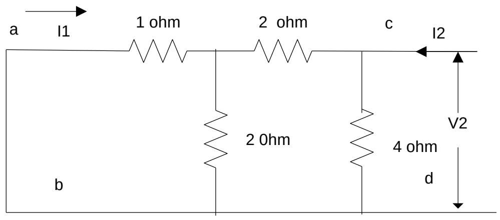


$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

$$Y_{11} = \frac{1}{1+\frac{2\times 2}{2+2}} = \frac{1}{2}mho$$

$$I_2 = \frac{-I_1 \times 2}{2} + 2 = \frac{-\frac{V_1}{2}\times 2}{2+2} = \frac{-V_1}{4}$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{I_2=0} = -\frac{1}{4}mho$$



$$I_2 = \frac{V_2}{\frac{\left[\frac{1\times 2}{1+2}+2\right]\times 4}{\left[\frac{1\times 2}{1+2}+2\right]+4}} + 2 = \frac{5V_2}{8}$$

$$Y_{22} = \frac{5}{8}mho$$

$$Y_{12} = -\frac{1}{4}mho$$

.

2.4.Transmission Parameters (ABCD parameters)

The transmission parameters A, B, C, and D express the required source variables V1 and I1 in terms
of the existing destination variables V2 and I2. They are called ABCD or T-parameters and are defined
by

$V_1 = AV_2 – BI_2$
$I_1= CV_2 – DI_2$

A $=V_1/V_2$ (for $I_2=0$)

B= - V_1/I_2 (for $V_2=0$)

C $=I_1/V_2$ (for $I_2=0$)

D = - I_1/I_2 (for $V_2=0$)

2.5.Hybrid parameters (lecture-6)

Short circuit and open circuit terminal conditions are used for determining the hybrid parameters. H – parameters representation used in modeling of electronic components and circuits.

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}, h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

Are called input impedance and forward current gain

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}, and\ h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

Are called reverse voltage gain and out put admittance

2.6.Condition for reciprocity and symmetry in two port parameters

A network is termed to be reciprocal if the ratio of the response to the excitation remains unchanged even if the positions of the response as well as the excitation are interchanged.
A two port network is said to be symmetrical it the input and the output port can be interchanged without altering the port voltages or currents.

In Z parameters

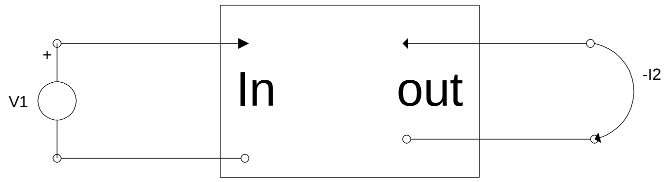


Fig.3

For short circuit and current direction output side is negative.

$V_1 = Z_{11} I_1 - Z_{12} I_2$

$0 = Z_{21} I_1 - Z_{22} I_2$

From the above two equations

$$I_2 = \frac{V_1 Z_{21}}{Z_{11} Z_{22} - Z_{12} Z_{21}}$$

Let us now interchange the input and output

$0 = -Z_{11} I_1 + Z_{12} I_2$

$V_2 = -Z_{21} I_1 + Z_{22} I_2$

From these equations we find

$$I_1 = \frac{V_2 Z_{12}}{Z_{11} Z_{22} - Z_{12} Z_{21}}$$

Assume

$V_1 = V_2$

Then

$Z_{12} = Z_{21}$

Symmetry in Z parameters

Keeping the output port open supplying V in the input side
We get,

$V = Z_{11} I_1$

$$Z_{11} = \frac{V}{I_1}$$

And applying voltage at the output port and keeping the input port open
We get,

$V = Z_{22} I_2$

$Z_{22} = \frac{V}{I_2}$ so condition of symmetry is achieved for

$Z_{22} = Z_{11}$

Similarly the other parameters can be studied and reported in table

parameter	Condition for reciprocity	Condition for symmetry

Z	$Z_{12} = Z_{21}$	$Z_{22} = Z_{11}$
Y	$Y_{12} = Y_{21}$	$Y_{11} = Y_{22}$
h	$h_{11} = -h_{21}$	$\Delta h = 1$
ABCD	$AD - BC = 1$	$A = D$

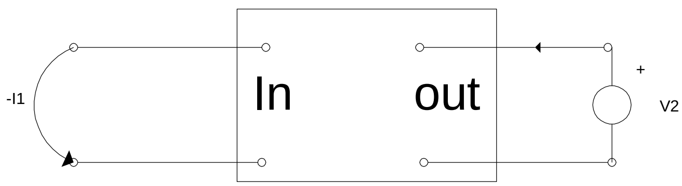


Fig.4

2.7.Interconnecting Two-Port Networks (lecture-7)

Two-port networks may be interconnected in various configurations, such as series, parallel, or cascade connection, resulting in new two-port networks. For each configuration, certain set of parameters may be more useful than others to describe the network.

2.7.1.Series Connection

A series connection of two two-port networks a and b with open-circuit impedance parameters Za and Zb, respectively. In this configuration, we use the Z-parameters since they are combined as a series connection of two impedances.

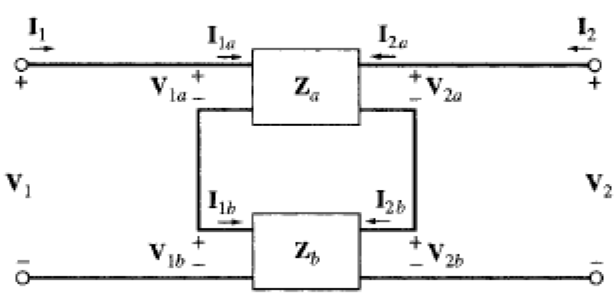


Fig.4

The Z-parameters of the series connection are

$Z_{11} = Z_{11A} + Z_{11B}$

$Z_{12} = Z_{12A} + Z_{12B}$

$Z_{21} = Z_{21A} + Z_{21B}$

$Z_{22} = Z_{22A} + Z_{22B}$

Or in the matrix form

$$[Z]=[Z_A]+[Z_B]$$

2.7.2.Parallel Connection
Fig.6

Fig.6

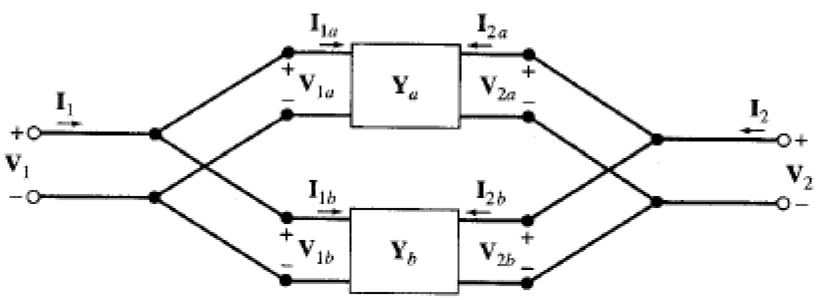


Fig.6

Figure shows a parallel connection of two-port networks a and b with short-circuit admittance parameters Ya and Yb. In this case, the Y-parameters are convenient to work with. The Y-parameters
of the parallel connection are (see Problem 13.11):

$Y_{11} = Y_{11A} + Y_{11B}$

Y_{12} ¼ $Y_{12A} + Y_{12B}$

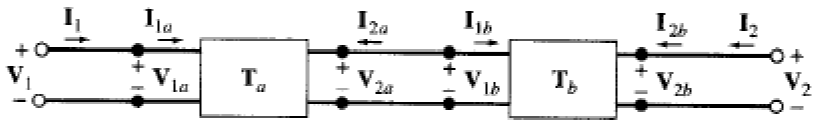
Y_{21} ¼ $Y_{21A} + Y_{21B}$

Y_{22} ¼ $Y_{22A} + Y_{22B}$

or, in the matrix form

$$[Y] = [YA] + [YB]$$

2.7.3.Cascade Connection



The cascade connection of two-port networks a and b is shown in Figure above. In this case the transmission-parameters are particularly convenient. The transmission-parameters of the cascade combination are

$A = A_a A_b + B_a C_b$
$B = A_a B_b + B_a D_b$
$C = C_a A_b + D_a C_b$
$D = C_a B_b + D_a D_b$

or, in the matrix form,

$$[T]=[T_a][T_b]$$

2.8. Y- parameters in terms of z- parameters (lecture-8)

$V_1 = Z_{11}I_1 + Z_{12}I_2$,
Or,
$$I_1 = \frac{(V_1 - Z_{12}I_2)}{Z_{11}}$$
similarly
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$
$$I_1 = \frac{(V_2 - Z_{22}I_2)}{Z_{21}}$$
Equating the two equations R.H.S
$$\frac{(V_1 - Z_{12}I_2)}{Z_{11}} = \frac{V_2 - Z_{22}I_2}{Z_{21}}$$
$$(V_1 - Z_{12}I_2)Z_{21} = (V_2 - Z_{22}I_2)Z_{11}$$
$$(V_1Z_{21} - Z_{12}Z_{21}I_2) = (V_2Z_{11} - Z_{22}Z_{11}I_2)$$
$$(Z_{22}Z_{11} - Z_{12}Z_{21})I_2 = (-V_1Z_{21} + V_2Z_{11})$$
$$I_2 = -\left(\frac{Z_{21}}{Z_{22}Z_{11} - Z_{12}Z_{21}}\right)V_1 + \left(\frac{Z_{11}}{Z_{22}Z_{11} - Z_{12}Z_{21}}\right)V_2$$

$$Y_{21} = \frac{-Z_{11}}{Z_{22}Z_{11} - Z_{12}Z_{21}}$$
$$Y_{22} = \frac{Z_{11}}{Z_{22}Z_{11} - Z_{12}Z_{21}}$$

Also from the two equations
$$I_2 = \frac{V_1 - Z_{11}I_1}{Z_{12}}$$
$$I_2 = \frac{V_2 - Z_{21}I_1}{Z_{22}}$$
And equating the both we get
$$\frac{V_1 - Z_{11}I_1}{Z_{12}} = \frac{V_2 - Z_{21}I_1}{Z_{22}}$$
$$(V_1 - Z_{11}I_1)Z_{22} = (V_2 - Z_{21}I_1)Z_{12}$$
$$(Z_{11}Z_{22} - Z_{21}Z_{12})I_1 = V_1Z_{22} - V_2Z_{12}$$
$$I_1 = \frac{Z_{22}}{(Z_{11}Z_{22} - Z_{21}Z_{12})}V_1 - \frac{Z_{12}}{(Z_{11}Z_{22} - Z_{21}Z_{12})}V_2$$
$$Y_{11} = \frac{Z_{22}}{(Z_{11}Z_{22} - Z_{21}Z_{12})}$$

$$Y_{12} = \frac{-Z_{12}}{D_{ZZ}}$$

Hence the y parameters can be expressed in z parameters as given below

$$\boxed{\begin{array}{l} Y_{11} =, \quad Y_{12} = \\ Y_{12} =, and \quad Y_{22} = \end{array}}$$

2.9.Z parameters in terms of Y parameters

$$\left.\begin{array}{l} I_1 = Y_{11}V_1 + Y_{12}V_2 \\ \\ \end{array}\right\} \qquad (2.9.1)$$

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$
From the above equation we find
$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$V_1 = \frac{I_1 - Y_{12}V_2}{Y_{11}}$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

$$V_1 = \frac{I_2 - Y_{22}V_2}{Y_{21}}$$

$$\frac{I_2 - Y_{22}V_2}{Y_{21}} = \frac{I_1 - Y_{12}V_2}{Y_{11}}$$

$$I_2Y_{11} - Y_{11}Y_{22}V_2 = Y_{21}I_1 - Y_{21}Y_{12}V_2$$

$$Y_{21}Y_{12}V_2 - Y_{11}Y_{22}V_2 = Y_{21}I_1 - I_2Y_{11}$$

$$(Y_{11}Y_{22} - Y_{21}Y_{12})V_2 = Y_{21}I_1 - I_2Y_{11}$$

$$V_2 = \frac{Y_{21}}{(Y_{11}Y_{22} - Y_{21}Y_{12})} I_1 - \frac{Y_{11}}{(Y_{11}Y_{22} - Y_{21}Y_{12})} I_2 \qquad (2.9.2)$$

$$V_2 = \frac{Y_{21}I_1 - I_2Y_{11}}{(Y_{11}Y_{22} - Y_{21}Y_{12})}$$

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \qquad (2.9.3)$$

$$V_2 = \frac{I_1 - Y_{11}V_1}{Y_{12}}$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \qquad (2.9.4)$$

$$V_2 = \frac{I_2 - Y_{21}V_1}{Y_{22}}$$

$$\frac{I_1 - Y_{11}V_1}{Y_{12}} = \frac{I_2 - Y_{21}V_1}{Y_{22}}$$

$$Y_{22}I_1 - Y_{22}Y_{11}V_1 = Y_{12}I_2 - Y_{12}Y_{21}V_1$$

$$(Y_{22}Y_{11} - Y_{12}Y_{21})V_1 = Y_{22}I_1 - Y_{12}I_2$$

$$V_1 = \frac{Y_{22}}{(Y_{22}Y_{11} - Y_{12}Y_{21})} I_1 - \frac{Y_{12}}{(Y_{22}Y_{11} - Y_{12}Y_{21})} I_2 \qquad (2.9.6)$$

$$Z_{11} = \frac{Y_{22}}{(Y_{22}Y_{11} - Y_{12}Y_{21})}$$

$$Z_{12} = \frac{Y_{12}}{(Y_{22}Y_{11} - Y_{12}Y_{21})}$$

Hence,

$$\boxed{\begin{aligned} Z_{11} &= \frac{Y_{22}}{\Delta Y} \quad , Z_{12} = \frac{Y_{12}}{\Delta Y} \\ Z_{21} &= \frac{Y_{21}}{\Delta Y} \quad , and Z_{22} = \frac{Y_{11}}{\Delta Y} \end{aligned}}$$

$$where, \Delta Y = Y_{11}Y_{22} - Y_{21}Y_{12}$$

2.10.Z parameters in terms of ABCD parameters

$$I_1 = CV_2 - DI_2$$
$$V_2 = \frac{1}{C}I_1 + \frac{D}{C}I_2$$
$$V_1 = AV_2 - BI_2$$
$$V_1 = \left(\frac{1}{C}I_1 + \frac{D}{C}I_2\right)A - BI_2$$
$$V_1 = \frac{A}{C}I_1 + \frac{AD - BC}{C}I_2$$

$$\boxed{\begin{aligned} Z_{11} &= \frac{A}{C} \quad , Z_{12} = \frac{AD - BC}{C} \\ Z_{21} &= \frac{1}{C} \quad , and Z_{22} = \frac{D}{C} \end{aligned}}$$

2.11.Z parameters in terms of hybrid- parameters

$$V_1 = h_{11}I_1 + h_{12}V_2 \qquad (2.11.1)$$
$$I_2 = h_{21}I_1 + h_{22}V_2 \qquad (2.11.2)$$

Using the above equation we get,

$$V_2 = -\frac{h_{21}}{h_{22}}I_1 + \frac{1}{h_{22}}I_2$$

Putting the value of V_2 in the first equation, we get,

$$V_1 = h_{11}I_1 + h_{12}\left(-\frac{h_{21}}{h_{22}}I_1 + \frac{1}{h_{22}}I_2\right)$$

$$V_1 = \left(\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}}\right) I_1 + \frac{h_{12}}{h_{22}} I_2$$

$$V_1 = \left(\frac{\Delta h}{h_{22}}\right) I_1 + \frac{h_{12}}{h_{22}} I_2$$

Comparing the equations with governing equation of Z-parameters

$$\boxed{\begin{array}{l} Z_{21} = -\dfrac{h_{21}}{h_{22}}, \quad Z_{22} = \dfrac{1}{h_{22}} \\ Z_{11} = \dfrac{\Delta h}{h_{22}}, \quad and Z_{12} = \dfrac{h_{12}}{h_{22}} \end{array}}$$

2.12.Y- parameters in terms of ABCD parameters

$$V_1 = AV_2 - BI_2 \qquad ()$$

$$I_2 = -\frac{1}{B} V_1 + \frac{A}{B} V_2 \qquad ()$$

$$I_1 = CV_2 - DI_2 \qquad ()$$

$$I_1 = CV_2 - D\left[\left(-\frac{1}{B}\right)V_1 + \frac{A}{B} V_2\right]$$

$$I_1 = \left[\left(\frac{D}{B}\right)V_1 - \frac{AD - BC}{B} V_2\right]$$

Hence,

$$\boxed{\begin{array}{l} Y_{11} = \dfrac{D}{B}, \quad Y_{12} = -\left(\dfrac{AD - BC}{B}\right) \\ Y_{21} = -\dfrac{1}{B}, and \ \ Y_{22} = \dfrac{A}{B} \end{array}}$$

2.13. ABCD parameters in terms of Y- parameters

The governing equation for Y parameters

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \tag{2.13.1}$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \tag{2.13.2}$$

Or,

$$V_1 = -\frac{Y_{22}}{Y_{21}}V_2 + \left(-\frac{1}{Y_{21}}\right) - I_2$$

For the above equation

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

Putting the value of V_1

$$I_1 = Y_{11}\left(-\frac{Y_{22}}{Y_{21}}V_2 + \left(-\frac{1}{Y_{21}}\right) - I_2\right) + Y_{12}V_2$$

$$I_1 = \left(-\frac{Y_{11}Y_{22} - Y_{21}Y_{12}}{Y_{21}}\right)V_2 - \left(-\frac{Y_{11}}{Y_{21}}\right)I_2$$

Comparing the equation with governing equation for ABCD parameters we get

$$\boxed{\begin{array}{l} A = -\dfrac{Y_{22}}{Y_{21}}, \quad B = -\dfrac{1}{Y_{21}} \\ C = -\dfrac{Y_{11}Y_{22} - Y_{21}Y_{12}}{Y_{21}}, andD = -\dfrac{Y_{11}}{Y_{21}} \end{array}}$$

3.Transients in DC circuits and AC circuits

3.1.What do you mean by transient? (lecture-9)

Whenever a circuit is switched from one condition to another, either by a change in the applied source or a change in the circuit elements, there is a transitional period during which the branch currents and element voltages change from their former values to new ones. This period is called the transient. After the transient has passed, the circuit is said to be in the steady state. Now, the linear differential equation that describes the circuit will have two parts to its solution, the complementary function (or the homogeneous solution) and the particular solution. The complementary function corresponds to the transient, and the particular solution to the steady state.

3.2.Transients in R-L circuit D.C and A.C source

3.2.1.Consider a series R-L circuit supplied by an DC source with a switch . Step response of dc circuit.

Take the initial condition of the circuit before closing the switch. Applying KVL to the given circuit. Taking laplace transform

$$RI(s) + L\{sI(s) - i(0^-)\} = \frac{V}{s}$$

$$(Ls + R)I(s) = \frac{V}{s}$$

$$I(s) = \frac{V}{s(Ls + R)}$$

$$I(s) = \frac{V/L}{s(s + R/L)}$$

$$= (\frac{A}{s} + \frac{B}{s + R/L})$$

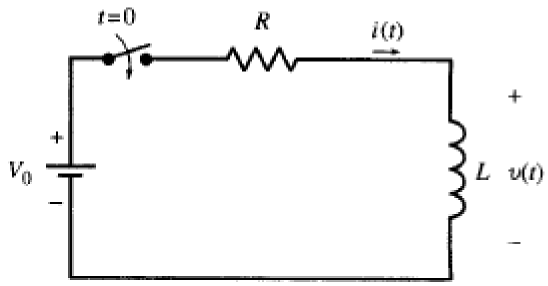
$$A = V/R$$

$$B = -V/R$$

$$I(s) = (\frac{V/R}{s} + \frac{-V/R}{s + R/L})$$

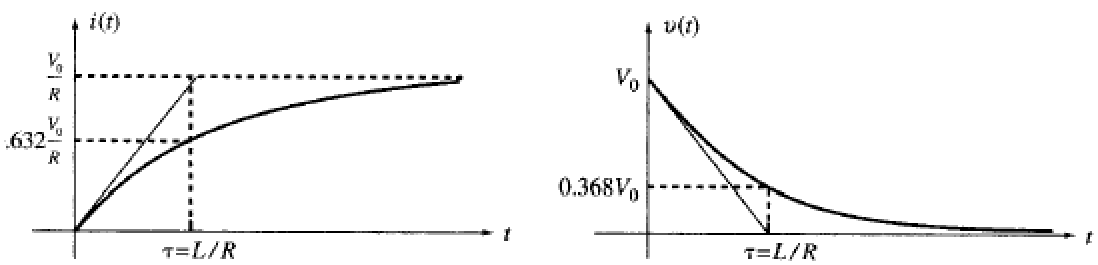
$$i(t) = \frac{V}{R} - \frac{V}{R} e^{-\frac{R}{L}t}$$

$$\boxed{i(t) = \frac{V}{R}(1 - e^{-\frac{R}{L}t})}$$



(a)

Fig.1



(b) (c)

3.2.2.Transient response of series R-L –C circuit with step input.

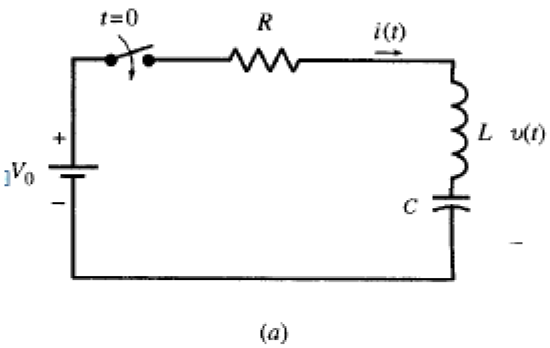


Fig.2.

$$I(s)=\frac{1/s}{R+Ls+1/Cs}$$

$$I(s)=\frac{1/L}{s^2+(R/L)s+1/LC}$$

$$I(s)=\frac{1/s}{(s+\alpha)(s+\beta)}$$

$$where \quad \alpha,\beta=-\frac{R}{2L}\pm\sqrt{\left(\frac{R}{2L}\right)^2-\frac{1}{LC}}$$

Case1

Both roots era real and not equal

$$\frac{R}{2L}>\frac{1}{LC}$$

$$I(s)=\frac{K_1}{(s+\alpha)}+\frac{K_2}{(s+\beta)}$$

$$K_1=\left.\frac{1/L}{(s+\beta)}\right|_{s=-\alpha}=\frac{1}{L(\beta-\alpha)}$$

$$K_2=\left.\frac{1/L}{(s+\alpha)}\right|_{s=-\beta}=\frac{1}{L(\alpha-\beta)}$$

$$I(s)=\frac{1}{L(\beta-\alpha)(s+\alpha)}+\frac{1}{L(\alpha-\beta)(s+\beta)}$$

$$i(t)=\left[\frac{1}{L(\beta-\alpha)}e^{-\alpha t}+\frac{1}{L(\alpha-\beta)}e^{-\beta t}\right]$$

Case 2

$$\frac{R}{2L}=\frac{1}{LC}$$

$$i(t) = \frac{1}{L} t e^{-\gamma t}$$

Case3

$$\frac{R}{2L} < \frac{1}{\sqrt{LC}}, \alpha = \beta *$$

Let

$$\alpha = -A_0 + jB *$$

$$\beta = -A_0 - jB *$$

$$I(s) = \frac{1/L}{(s + A_0 - jB)(s + A_0 + jB)}$$

$$I(s) = \frac{K_3}{(s + A_0 - jB)} + \frac{K_3^*}{(s + A_0 + jB)}$$

$$K_3 = (s + A_0 - jB) I(s) \Big|_{s=-A_0+jB}$$

$$K_3 = \left. \frac{1/L}{(s + A_0 + jB)} \right|_{s=-A_0+jB} = \frac{1/L}{j2B}$$

$$K_3^* = -\frac{1/L}{j2B}$$

So,

$$I(s) = \frac{\frac{1/L}{j2B}}{(s + A_0 - jB)} + \frac{(-\frac{1/L}{j2B})}{(s + A_0 + jB)}$$

Taking inverse Laplace transform

We get,

$$i(t) = \frac{1}{j2BL} e^{-A_0 t} e^{jBt} - \frac{1}{j2BL} e^{-A_0 t} e^{-jBt}$$

Or,

$$i(t) = \frac{e^{-A_0 t}}{BL} \left(\frac{e^{jBt} - e^{-jBt}}{j2} \right)$$

$$i(t) = \frac{e^{-A_0 t}}{BL} \left(\sin Bt \right)$$

3.2.3.Pulse response of RC series circuit (lecture-10)

In this section we will derive the response of a first-order circuit to a rectangular pulse. The derivation applies to RC or RL circuits where the input can be a current or a voltage. As an example, we use the series RC circuit in Fig. 7-17(a) with the voltage source delivering a pulse of duration T and height V0. For t < 0, v and i are zero. For the duration of the pulse, we use (6b) and (6c) in Section

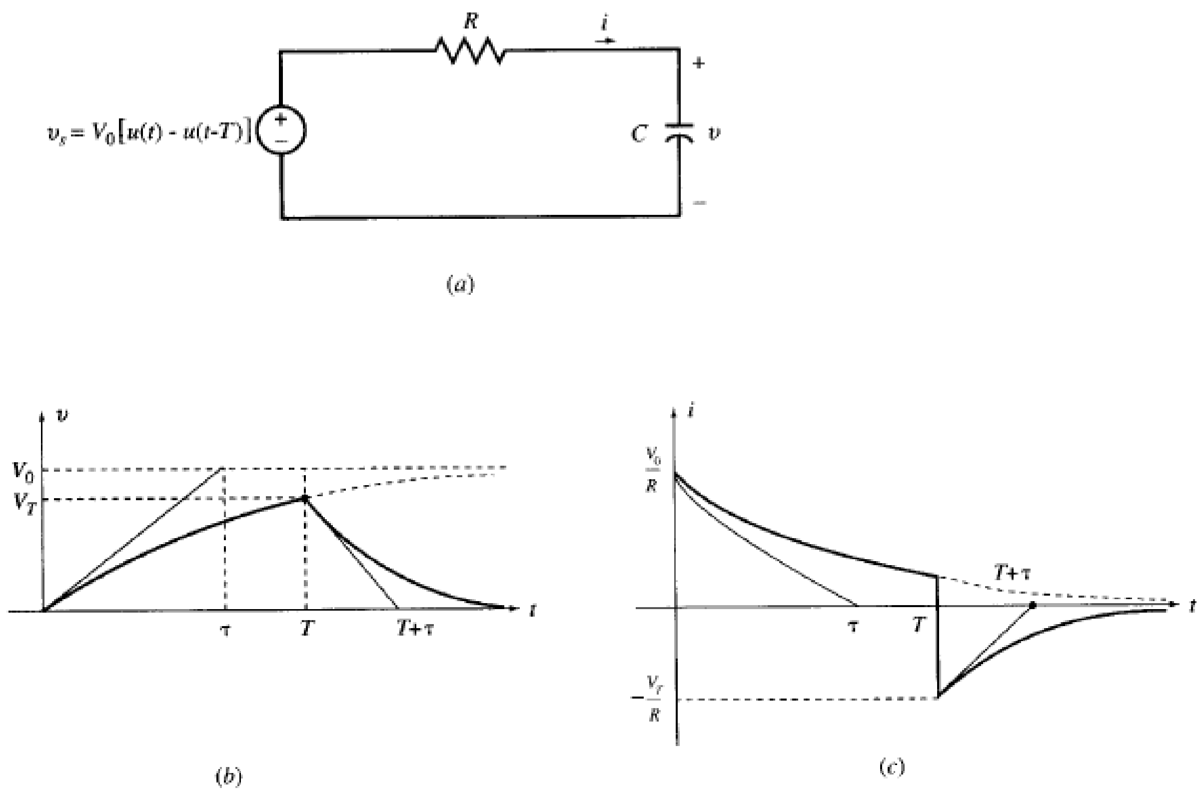


Fig.3

$$v = V_0(1 - e^{-t/RC}) \qquad 0 < t < T$$

$$i = \frac{V_0}{R} e^{-t/RC} \qquad 0 < t < T$$

When the pulse ceases, the circuit is source-free with the capacitor at an initial voltage V_T .

$$V_T = V_0(1 - e^{-T/RC}) \qquad t > T$$

taking into account the time shift T, we have

$$v = V_T(1 - e^{-(t-T)/RC}) \qquad t > T$$

$$i = -\frac{V_T}{R} e^{-(t-T)/RC} \qquad t > T$$

The capacitor voltage and current are plotted in Figs.(b) and (c)

3.2.4.R-L series with sine wave

Applying KVL to the circuit

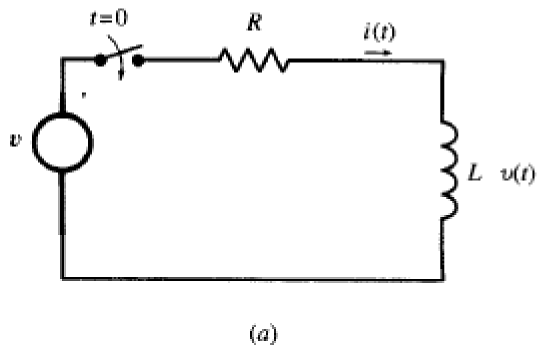


Fig.4.

Applying KVL to the circuit

$$i(t)R + L\frac{di}{dt} = E_m \sin \omega t$$

Taking Laplace transform

$$I(s)R + LsI(s) - sI(0) = E_m\left(\frac{\omega}{s^2 + \omega^2}\right)$$

Or

$$I(s)R + LsI(s) - 0 = E_m\left(\frac{\omega}{s^2 + \omega^2}\right)$$

$$I(s)(R+Ls)=E_m\left(\frac{\omega}{s^2+\omega^2}\right)$$

$$I(s)=\frac{E_m}{(R+Ls)}\left(\frac{\omega}{s^2+\omega^2}\right)$$

Or

$$I(s)=\frac{E_m}{L(s+R/L)}\left(\frac{\omega}{(s+j\omega)(s-j\omega)}\right)$$

$$I(s)=\frac{K_1}{(s+R/L)}+\frac{K_2}{(s+j\omega)}+\frac{K_3}{(s-j\omega)}$$

$$K_1=\left[\frac{E_m}{L(s+R/L)}\left(\frac{\omega}{(s+j\omega)(s-j\omega)}\right)\right]\Bigg|_{s=-R/L}=\frac{E_m\omega}{R^2+\omega^2L^2}$$

$$K_2=\left[\frac{E_m}{L(s+R/L)}\left(\frac{\omega}{(s+j\omega)(s-j\omega)}\right)\right]\Bigg|_{s=-j\omega}=\frac{E_m}{-2j(R-j\omega L)}$$

$$K_3=\left[\frac{E_m}{L(s+R/L)}\left(\frac{\omega}{(s+j\omega)(s-j\omega)}\right)\right]\Bigg|_{s=j\omega}=\frac{E_m}{2j(R-j\omega L)}$$

Thus

$$I(s)=\frac{E_m\omega L}{(R^2+\omega^2L^2)(s+R/L)}+\frac{E_m}{-2j(R-j\omega L)(s+j\omega)}+\frac{E_m}{2j(R-j\omega L)(s-j\omega)}$$

Taking inverse Laplace Transform

$$\boxed{i(t)=\frac{E_m\omega L}{(R^2+\omega^2L^2)}e^{-Rt/L}-\frac{E_m(R+j\omega L)}{2j(R^2+\omega^2L^2)}e^{-j\omega t}+\frac{E_m(R-j\omega L)}{2j(R^2-j\omega^2L^2)}e^{j\omega t}}$$

NETWORK FUNCTIONS & RESPONSES

4.1 INTRODUCTION(lecture-11)

In engineering, a transfer function (also known as the system function or network function and, when plotted as a graph, transfer curve) is a mathematical representation for fit or to describe the inputs and outputs of black box models.

All the systems are designed to produce a particular output, when the input is applied to it. The system parameters, perform some operation on the applied input, in order to produce the required output. The mathematical indication of cause and effect relationship existing between input and output of a system is called the system function or transfer function of the system. The Laplace transform plays an important role in defining the system function.

4.2 CONCEPT OF COMPLEX FREQUENCY

A complex number used to characterize exponential and damped sinusoidal motion in the same way that an ordinary frequency characterizes the simple harmonic motion; designated by the constants corresponding to a motion whose amplitude is given by Ae^{st}, where A is a constant and *t* is time.

A type of frequency that depends on two parameters; one is the "σ" which controls the magnitude of the signal and the other is "ω", which controls the rotation of the signal; is known as "complex frequency".

A complex exponential signal is a signal of type 1

$$X(t) = X_m e^{st} \text{……………………………...} \quad (4.2.1)$$

Where X_m and s are time independent complex parameter and S = σ + jω where X_m is the magnitude of X (t), sigma (σ) is the real part in S and is called neper frequency and is expressed in Np/s. "ω" is the radian frequency and is expressed in rad/sec. "S" is called complex frequency and is expressed in complex neper/sec.

Now put the value of S in equation (4.2.1), we get

$$X(t) = X_m e^{\sigma t + j\omega t}$$

$$X(t) = X_m e^{\sigma t} e^{j\omega t}$$

By using Euler's theorem

i.e. $e^{i\theta} = \cos\theta + i \sin\theta$

$$X(t) = X_m e^{\sigma t}[\mathrm{Cos}\ (\omega t) + j \sin\ (\omega t)]$$

The real part is

$$X\ (t) = X_m e^{\sigma t} \cos(\omega t)$$

And imaginary part is

$$X\ (t) = X_m e^{\sigma t} \sin(\omega t)$$

The physical interpretation of complex frequency appearing in the exponential form will be studied easily by considering a number of special cases for the different value of S.

Case no 1: When ω =0 and σ has a certain value, then, the real part is

$$X\ (t) = X_m e^{\sigma t} \cos(\omega t)$$

$$X\ (t) = X_m e^{\sigma t}.1$$

$$X\ (t) = X_m e^{\sigma t}$$

The imaginary part is zero (0),
Since S = σ + jω, S =σ as ω = 0

Now there are also three cases in above case no 1.

(i) If the neper frequency is positive i.e. σ > 0 the curve obtains is exponentially increasing curve as shown below.

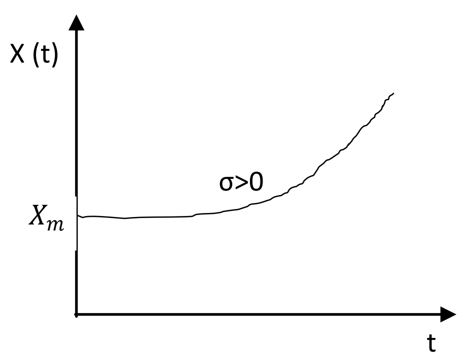


Fig. 4.1

(ii) If σ < 0 then the curve obtain is exponentially decreasing curve as shown below.

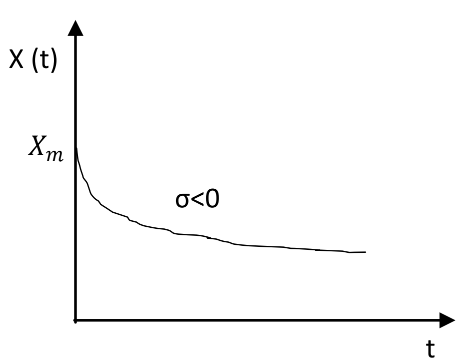


Fig. 4.2

(iii) If σ = 0 then the curve obtain is the steady state DC curve as shown below fig. 4.3

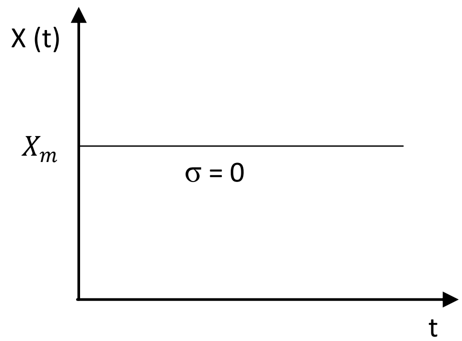


Fig. 4.3

Case no 2: When σ = 0 and ω has some value then, the real part is

$$X(t) = X_m e^{0.t} \cos(\omega t)$$

$$X(t) = X_m \cos(\omega t)$$

And the imaginary part is

$$X(t) = X_m \sin(\omega t)$$

Hence the curve obtained is a sinusoidal steady state curve, as shown in the below figure.

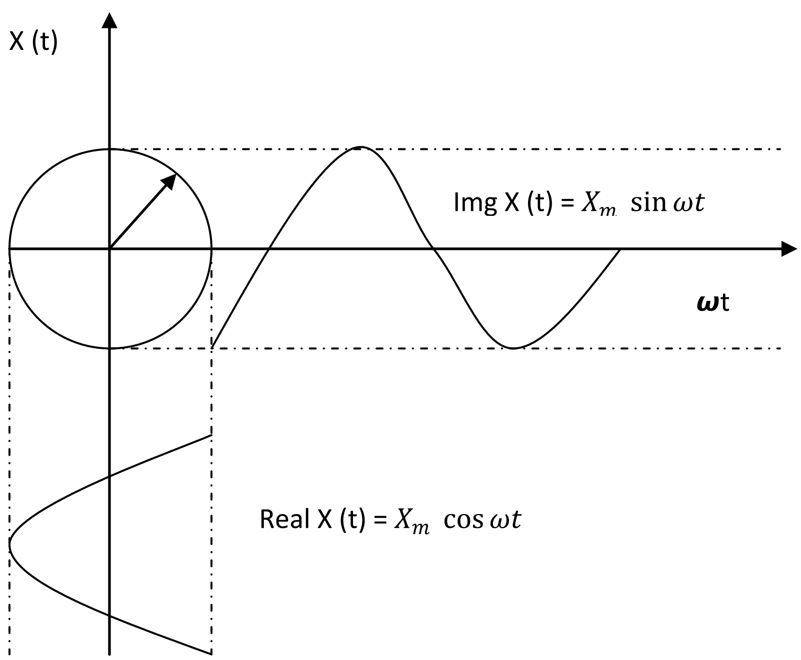


Fig. 4.4

Case no 3: When σ and ω both have some value, then the real part is

$$X(t) = X_m e^{\sigma t} \cos(\omega t)$$

And the imaginary part is

$$X(t) = X_m e^{\sigma t} \sin(\omega t)$$

So, the curve obtained is time varying sinusoidal signal. These case no 3 is also has some two cases

(a) When σ > 0

Real Part

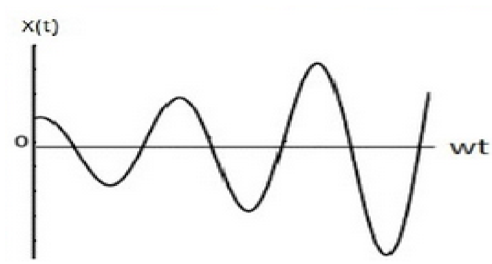


Fig. 4.5

Imaginary Part

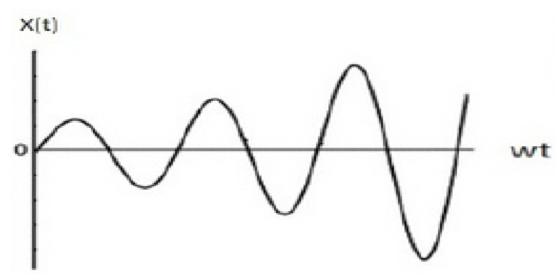


Fig. 4.6

(b) When σ< 0

Real Part

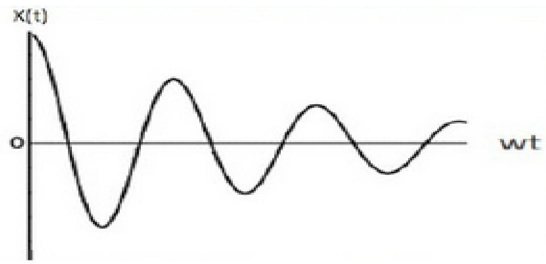


Fig. 4.7

Imaginary Part

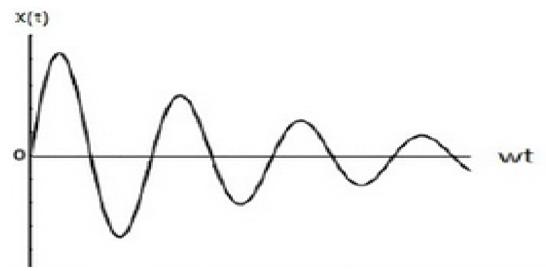


Fig. 4.8

4.3 NETWORK FUNCTIONS FOR ONE PORT NETWORK: (Lecture -12)

(a) Driving Point Impedance Function

The ratio of the Laplace transform of the voltage at the port to the Laplace transform of the current at the same port, neglecting initial conditions is called driving point impedance function, denoted as Z(s)

$$Z(s) = \frac{V(s)}{I(s)}$$

(b) Driving Point Admittance Function

The ratio of the Laplace transform of the current at the port to the Laplace transform of the voltage at the same port, neglecting initial conditions is called driving point admittance function.

$$Y(s) = \frac{I\ (s)}{V\ (s)}$$

Example 4.3.1 For the given one port network shown in fig., find driving point impedance function.

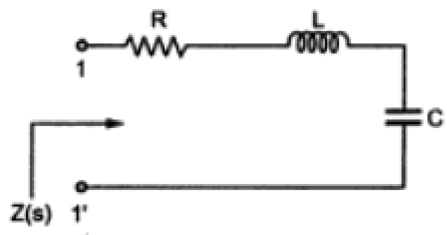


Fig.4.9

Solution: To find the driving point impedance for the given network, first convert the network into s domain which is called transformed network.

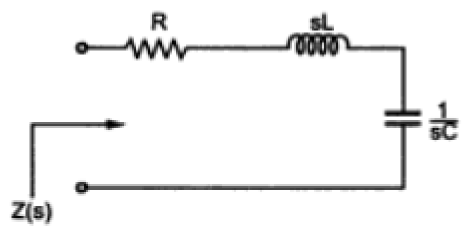


Fig. 4.10

The transformed network is as shown in fig

Then, $$Z(s) = R + sL + \frac{1}{sC}$$

$$= \frac{R(sC)+(sL)(sC)+1}{sC} = \frac{sRC+s^2LC+1}{sC}$$

Hence, the driving point impedance function of a given network is

$$Z(s) = \frac{s^2LC+sRC+1}{sC}$$

Example 4.3.2 Find driving point admittance function for the given network having only one port.

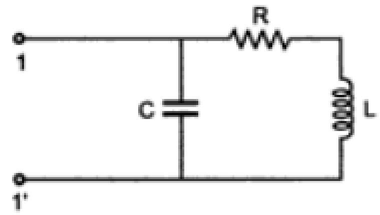


Fig. 4.11

Solution: First transform the given network as shown in fig. 4.12

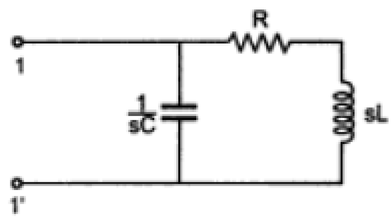


Fig. 4.12 Transformed network

When the branches are connected in parallel it is very easy to find admittance of an overall function by adding admittances of individual branches which are connected in parallel.

Admittance of capacitive branch is given as

$$Y_1(s) = \frac{1}{1/sC} = sC$$

Admittance of the inductive branch is given as

$$Y_2(s) = \frac{1}{R + sL}$$

∴Total admittance is given as

$$Y(s) = Y_1(s) + Y_2(s)$$

$$= sC + \frac{1}{R + sL} = \frac{sRC + s^2LC + 1}{R + sL}$$

Hence, the driving point admittance function of a given network is,

$$Y(s) = \frac{s^2LC + sRC + 1}{R + sL}$$

4.4 NETWORK FUNCTIONS FOR TWO PORT NETWORK: (Lecture -13)

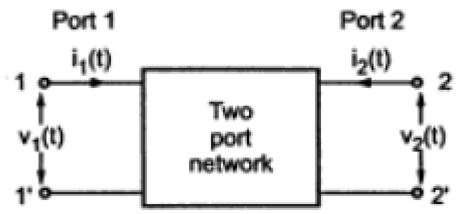


Fig. 4.13

For the two port network, there are two ports. The variables measured at port 1 are $v_1(t)$ and $i_1(t)$ while the variables measured at port 2 are $v_2(t)$ and $i_2(t)$. This is shown in above fig. 4.13. The ratio of the variables measured at the same port either port 1 or port 2 defines driving point function. Hence, for two port networks, we can define,

(a) Driving Point Impedance Function

This is the ratio of $V_1(s)$ and $I_1(s)$ at port 1, denoted as $Z_{11}(s)$ or the ratio of $V_2(s)$ to $I_2(s)$ at port 2, denoted as$Z_{22}(s)$. Both are driving impedance functions.

$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)} \text{ and } Z_{22}(s) = \frac{V_2(s)}{I_2(S)}$$

(b) Driving Point Admittance Function

The reciprocals of the driving point impedance functions at the ports give the driving point admittance functions at the respective ports. These are denoted as $Y_{11}(s)$ and $Y_{22}(s)$ at the two ports respectively.

$$Y_{11}(s) = \frac{I_1(s)}{V_2(s)} \text{and } Y_{22}(s) = \frac{I_2(s)}{V_2(s)}$$

4.5 TRANSFER FUNCTION OF TWO PORT NETWORK:

The function defined as the ratio or Laplace of two variables such that one variable is defined at one port while the other variable is defined at the second port. Accordingly, four transfer functions can be defined for two port network as,

1. Voltage Ratio Transfer Function

It is the ratio of Laplace transforms of voltage at one port to voltage at other port. It is denoted as G(s).

$$\therefore G(s) = \frac{V_2(s)}{V_1(s)}$$

2. Current Ratio Transfer Function

It is the ratio of Laplace transforms of current at one port to current at other port. It is denoted as α(s).

$$\therefore \alpha(s) = \frac{I_2(s)}{I_1(s)} \ or \ \frac{I_1(s)}{I_2(s)}$$

3. Transfer Impedance Function

It is the ratio of Laplace transforms of voltage at one port to current at other port. It is denoted as $Z_{12}(s)$ or $Z_{21}(s)$.

$$\therefore Z_{21}(s) = \frac{V_2(s)}{I_1(s)} \ and \ Z_{12}(s) = \frac{V_1(s)}{I_2(s)}$$

4. Transfer Admittance Function

It is the ratio of Laplace transforms of current at one port to voltage at other port. It is denoted as $Y_{12}(s)$ or $Y_{21}(s)$.

$$\therefore Y_{21}(s) = \frac{I_2(s)}{V_1(s)} \ and \ Y_{12}(s) = \frac{I_1(s)}{V_2(s)}$$

Example 4.5.1 For given two port network find driving point impedance function and voltage ratio transfer function.

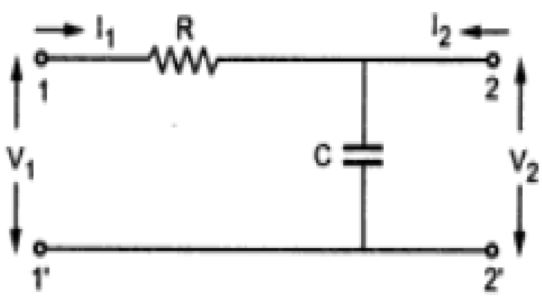


Fig. 14

Solution: Transforming given network in s domain, the network will be as shown in below fig.

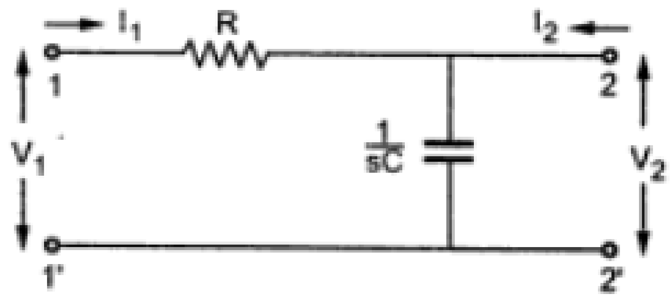


Fig 15

To find voltage ratio transfer function, find V_2 using potential divider rule as follows,

$$V_2 = V_1\left[\frac{1/sC}{R+1/sC}\right]$$

$$\therefore \frac{V_2}{V_1} = \frac{1/sC}{\frac{sRC+1}{sC}}$$

$$\frac{V_2}{V_1} = \frac{1}{sRC+1}$$

As port 2 is open circuited, $I_2 = 0$.

Hence current I_1 will flow through R and C.

$$V_1 = I_1\left[R+\frac{1}{sC}\right]$$

$$Z(s) = \frac{V_1}{I_1} = \frac{sRC+1}{sC}$$

4.6 POLES & ZEROS OF NETWORK FUNCTIONS : (lecture-14)

In pole/zero analysis, a network is described by its network transfer function which, for any linear time-invariant network, can be written in the general form:

$$H(s) = \frac{N(s)}{D(s)} = \frac{a_0 s^m + a_1 s^{(m-1)} + \cdots + a_m}{b_0 s^n + b_1 s^{(n-1)} + \cdots + b_n}$$

In the factorized form, the general function is:

$$H(s) = \frac{a_0}{b_0} \cdot \frac{(s+z_1)(s+z_2)\ldots(s+z_i)\ldots(s+z_m)}{(s+p_1)(s+p_2)\ldots(s+p_j)\ldots(s+p_n)}$$

The roots of the numerator N(s) (that is, z_i) are called the zeros of the network function, and the roots of the denominator D(s) (that is, p_j) are called the poles of the network function. S is a complex frequency.

The dynamic behavior of the network depends upon the location of the poles and zeros on the network function curve. The poles are called the natural frequencies of the network. In general, you can graphically assume the magnitude and phase curve of any network function from the location of its poles and zeros.

Example 4.6.1 A function given by $Z(s) = \frac{S+4}{S}$. Find the pole-zero plot.

Solution: In the above function the zero is at s = -4 and the pole is at s = 0. The pole-zero plot is shown below in fig. 16

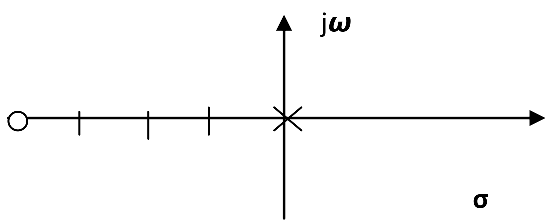


Fig. 16

Example 4.6.2 A function given by $Z(s) = \frac{2s}{(s+2)(s^2+2s+2)}$. Obtain its pole-zero plot.

Solution: The zero is at s = 0 and the pole is at s = -2,(-1+j1),(-1-j1) . The pole-zero plot is shown below in fig. 17

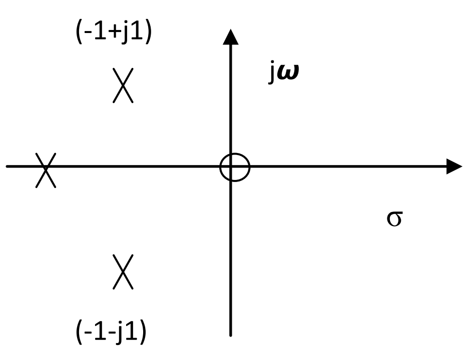


Fig. 17

4.7 RESTRICTION ON POLE AND ZERO LOCATIONS OF NETWORK FUNCTION

(Lecture -15)

The following are the restrictions on pole and zero location of network function:

(a) The co-efficient of the polynomials of numerator and denominator of the network function H (s) must be real and positive.
(b) Poles and zeros, if complex or imaginary, must occur in conjugate pairs.
(c) The real parts of all poles and zeros must be zero or negative.
(d) The polynomial of the numerator or denominator cannot have any missing term between those of highest and lowest order values unless all even order or all odd order terms are missing.
(e) The degree of the numerator or the denominator polynomial may differ by zero or one only.
(f) The lowest degree in numerator and denominator may differ in degree by at the most one.

Example 4.7.1 Check if the impedance function Z(s) given by $\boldsymbol{Z(s) = \frac{s^4+s^2+1}{s^3+2s^2-2s+10}}$ can represent a passive one port network.

Solution: The given function is not suitable to represent the impedance of a one port network due to the following reasons:

(i) In the numerator one co-efficient is missing.

(ii) In the denominator one co-efficient is negative.

4.8 IMPULSE RESPONSE

The system function is defined as,

$$H(s) = \frac{Laplace\ \ of\ output\ \ response}{Laplace\ \ of\ input\ \ excitation} = \frac{R(s)}{E(s)}$$

$$\therefore R(s) = H(s).E(s)$$

Let the input impulse function $\delta(t)$.

$$\therefore e(t) = \delta(t)$$

$$\therefore E(s) = L\{\delta(t)\} = 1$$

$$\therefore R(s) = H(s).1 = H(s)$$

Thus the Laplace of the response is same as the system function, for unit impulse input. Thus, if r (t) is the impulse response of the network then as

$$R(s) = H(s)$$

$$\text{Impulse response } = L^{-1}[R(s)] = L^{-1}[H(s)] = h(t)$$

Thus $\boldsymbol{h(t) = L^{-1}[H(s)]}$ is nothing but the impulse response of the network.

Example 4.8.1 Find the impulse response of the network shown in below fig. 16. The excitation is the voltage v (t) while the response is i (t).

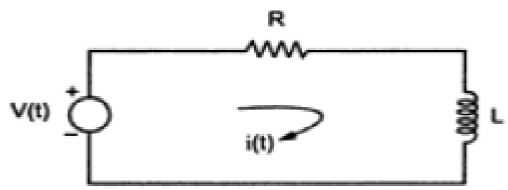


Fig. 16

Solution: The excitation is v(t) and i(t) is response hence,

$$H(s) = \frac{I(s)}{V(s)}$$

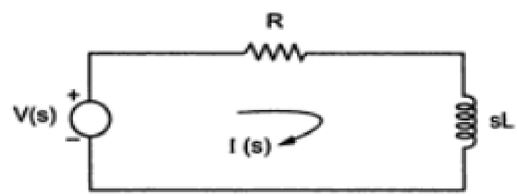


Fig. 17

As the initial condition is zero, the circuit can be transformed in the Laplace domain as shown in above fig.

After applying KVL,

$$-I(s)\,R - I(s)\,Ls + V(s) = 0$$

$$I(s)\,[R + Ls] = V(s)$$

$$H(s) = \frac{I(s)}{V(s)} = \frac{1}{R + sL}$$

Example 4.8.2 Impulse response of a circuit is given as $h(t) = -te^{-t} + 2e^{-t}, t > 0$. Find the transfer function of the network.

Solution: L [Impulse Response] = Transfer Function

$$H(s) = -\frac{1}{(s+1)^2} + \frac{2}{(s+1)}$$

$$= \frac{-1+2(s+1)}{(s+1)^2}$$

$$= \frac{2s+1}{(s+1)^2}$$

Example 4.8.3 The transfer function of a network is given by $(s) = \frac{1}{1+sRC}$. Find its impulse response.

Solution: $\delta(t) = L^{-1}\left[\frac{1}{1+sRC}\right]$

$$= L^{-1}\left[\frac{\frac{1}{RC}}{\frac{1}{RC}+s}\right]$$

$$= \frac{1}{RC}e^{-t/RC}$$

4.9 COMPLETE RESPONSE

Complete response is the sum of forced response (steady state response) and source free response (natural response). The response of a circuit or network with presence of source is called forced or source response. This response is independent of the nature of passive elements and their initial condition but this response completely depends on type of input. This response is different for different types of input. Response of any circuit or network without any source is called as source free response or natural response.

Example 4.9.1 For the circuit given below, find complete response for ***i(t)*** if v (0) =15 V.

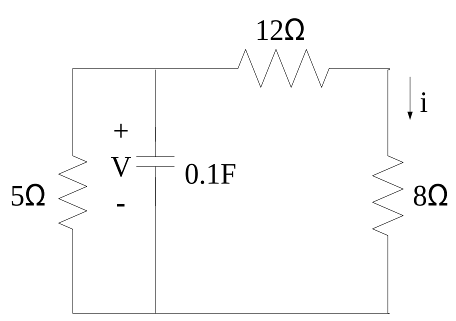


Fig.18

Solution: It is natural response due to the capacitor internal stored energy. This is a source free first order RC circuit.

$$v(t) = v_o e^{-\frac{t}{\tau}}$$

$$R_{eq} = 5||20 = 4\Omega$$

$$\tau = RC = 4 * 0.1 = 0.4sec$$

$$v(t) = 15e^{\frac{-t}{0.4}} = 15e^{-2.5t}V$$

$$i(t) = \frac{v(t)}{R} = \frac{15e^{-2.5t}}{12+8} = 0.75e^{-2.5t}A$$

4.10 TIME DOMAIN BEHAVIOR FROM POLE-ZERO PLOT (Lecture -16)

In time-domain analysis the response of a dynamic system to an input is expressed as a function of time. It is possible to compute the time response of a system if the nature of input and the mathematical model of the system are known.

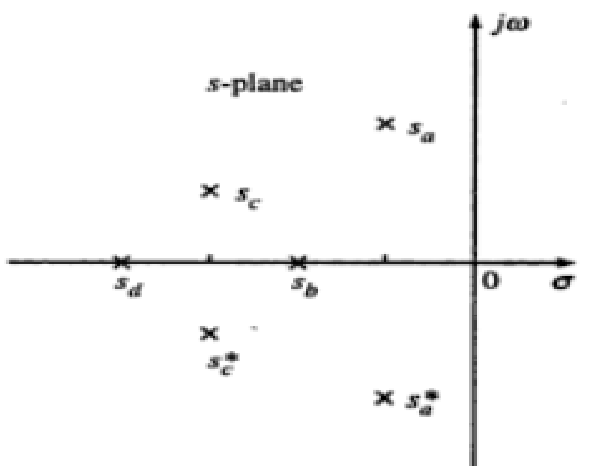


Fig.19 Array of poles

The poles s_b and s_d are quite different from the conjugate pole pairs. Further, s_b and s_d are the real poles and the response due to the poles s_b and s_d converges monotonically. The poles s_b and s_d may correspond to the quadratic function,

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0; \zeta > 1$$

And the roots

$$s_b, s_d = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}; \quad \zeta > 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The contribution to the total response, due to poles s_b and s_d is

$$K_b e^{s_b t} + K_d e^{s_d t}$$

The contribution to response due to the pole s_b is predominant compared to that of s_d as $|s_d| \gg |s_b|$. In this case, s_b is dominant compared to s_d. The pole s_b is the dominant pole amongst these two real poles. The response due to pole at s_d dies down faster compared to that due to pole at s_b. The complex conjugate poles s_a and s_a^* which belong to the quadratic factor

$$s^2 + 2\zeta\omega_n s + \omega_n^2; \quad \zeta > 1$$

The roots are

$$s_a, s_a^* = -\zeta\omega_n \pm \mathrm{j}\omega_n\sqrt{1-\zeta^2}$$

Contribution to the total response by the complex conjugate poles s_a and s_a^* is

$$K_a e^{-\zeta\omega_n t} e^{\mathrm{j}\omega_n\sqrt{1-\zeta^2}t} + K_a^* e^{-\zeta\omega_n t} e^{-\mathrm{j}\omega_n\sqrt{1-\zeta^2}t}$$

The factor $\exp(-\zeta\omega_n t)$ gives the decreasing function, whereas the factor $\exp(\mathrm{j}\omega_n\sqrt{1-\zeta^2}t)$ gives sustained oscillation. The resultant will give the damped sinusoid waveform, as shown in fig. 20

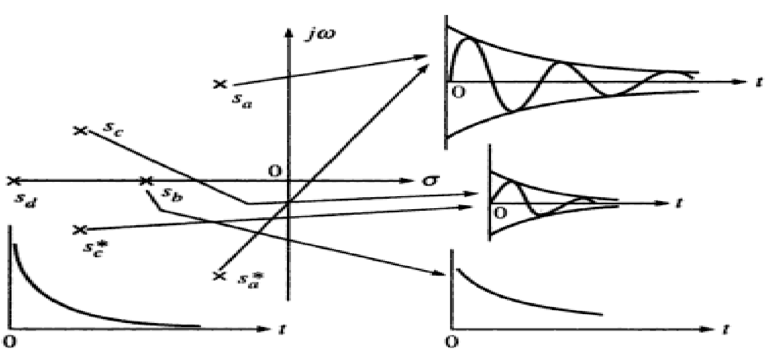


Fig. 20 Nature of response for arbitrary magnitude for all poles.

Similarly, s_c and s_c^* are also the complex conjugate pole pair. The response due to s_c and s_c^* will die down faster than due to s_a and s_a^* . Hence s_a and s_a^* are the dominant complex conjugate pole pair compared to the complex pole pair s_c and s_c^*. The time domain response can be obtained by taking the inverse Laplace transform after the partial fraction expansion,

$$\mathcal{L}^{-1}\left[\frac{K_a}{s-s_a} + \frac{K_a^*}{s-s_a^*} + \frac{K_b}{s-s_b} + \frac{K_c}{s-s_c} + \frac{K_c^*}{s-s_c^*} + \frac{K_d}{s-s_d}\right]$$

Where the residues K_a and K_a^* are complex conjugate, as also the residues K_c and K_c^*. Any residues K_r can be obtained as

$$K_r = H \times \frac{(s - s_1) \dots (s - s_n)}{(s - s_a) \dots . (s - s_r) \dots (s - s_m)} (s - s_r)$$

The term $(s_r - s_n)$ is also a complex number expressed in polar co-ordinates as

$$s_r - s_n = M_{rn} \exp(j\phi_{rn})$$

$$K_r = H \times \frac{M_{r1} \dots . . M_{rn}}{M_{ra} \dots \dots M_{rm}} expj(\phi_{r1} + \phi_{r2} \dots . . \phi_{rn} - \phi_{ra} \dots \phi_{rm})$$

Example 4.10.1 A transfer function is given by $Y(s) = \frac{10S}{(S+5+J15)(S+5-J15)}$.Find the time domain response.

Solution: $Y(s) = k1e^{-(5+j15)t} + k2e^{-(5-j15)t}$

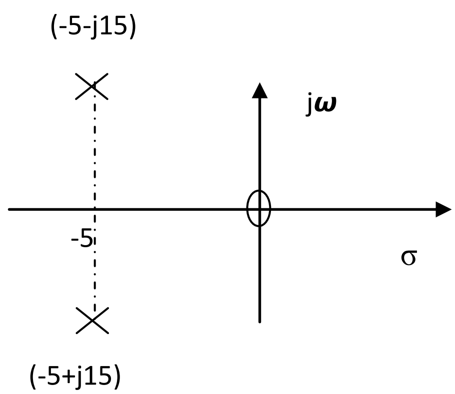


Fig.21

$$k1 = \frac{10\sqrt{5^2 + 15^2} < 90 + tan^{-}\left(\frac{5}{15}\right)}{30 < 90} = 5.267 < 18.4$$

$$k2 = \frac{10\sqrt{5^2 + 15^2} < -90 - tan^{-}\left(\frac{5}{15}\right)}{30 < -90} = 5.267 < -18.4$$

$$Y(t) = 5.267 < 18.4e^{-(5+j15)t} + 5.267 < -18.4e^{-(5-j15)t}$$

THREE PHASE CIRCUITS

5.1 INTRODUCTION (Lecture -17)

There are two types of system available in electric circuit, single phase and **three phase system**. In single phase circuit, there will be only one phase, i.e. the current will flow through only one wire and there will be one return path called neutral line to complete the circuit. So in the single phase minimum amount of power can be transported. Here the generating station and load station will also be single phase. This is an old system using from previous time.

In 1882, a new invention has been done on polyphase system, that more than one phase can be used for generating, transmitting and for load system. **Three phase circuit** is the polyphase system where three phases are sent together from the generator to the load. Each phase is having a phase difference of 120°, i.e. 120° angle electrically. So from the total of 360°, three phases are equally divided into 120° each. The power in **three phase system** is continuous as all the three phases are involved in generating the total power. The sinusoidal waves for 3 phase system is shown below

The three phases can be used as single phase each. So if the load is single phase, then one phase can be taken from the **three phase circuit** and the neutral can be used as ground to complete the circuit.

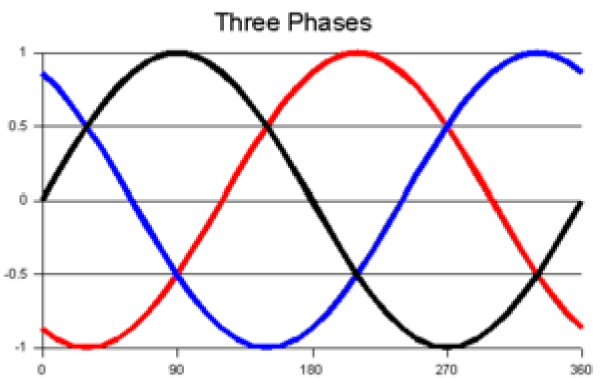


Fig. 1

5.2 IMPORTANCE OF THREE PHASE OVER SINGLE PHASE

There are various reasons for this because there are a number of advantages over single phase circuit. The three phase system can be used as three single phase line so it can act as three single phase system. The three phase generation and single phase generation are the same in the generator except the arrangement of coil in the generator to get 120° phase difference. The conductor needed in three phase circuit is 75% that of conductors needed in a single phase circuit. And also the instantaneous power in a single phase system falls down to zero as in single phase, we can see from the sinusoidal curve, but in three phase system the net power from all the phases gives a continuous power to the load.

Till now we can say that there are three voltage source connected together to form a three phase circuit. And actually it is inside the generator. The generator is having three voltage sources which are acting together in 120° phase difference. If we can arrange three single phase circuit with 120° phase difference, then it will become a three phase circuit. So 120° phase difference is must otherwise the circuit will not work, the three phase load will not be able to get active and it may also cause damage to the system.

The size or metal quantity of three phase devices is not having much difference. Now if we consider the transformer, it will be almost same size for both single phase and three phase because transformer will make only the linkage of flux. So the three phase system will have a higher efficiency compared to single phase because of the same or little difference in mass of transformer, three phase line will be out whereas in single phase it will be only one. And losses will be minimized in three phase circuit. So overall in conclusion the three phase system will have better and higher efficiency compared to the single phase system.

In a three phase circuit, connections can be given in two types:

1. Star connection
2. Delta connection

5.2.1 STAR CONNECTION

In star connection, there are four wires, three wires are phase wire and fourth is neutral which is taken from the star point. Star connection is preferred for long distance power transmission because it is having the neutral point. In this we need to come to the concept of balanced and unbalanced current in power system.

When equal current will flow through all the three phases, then it is called as balanced current. And when the current will not be equal in any of the phases, then it is unbalanced current. In this case, during balanced condition, there will be no current flowing through the neutral line and hence there is no use of the neutral terminal. But when there will be an unbalanced current flowing in the three phase circuit, neutral is having a vital role. It will take the unbalanced current through to the ground and protect the transformer.

Unbalanced current affects transformer and it may also cause damage to the transformer and for this star connection is preferred for long distance transmission. The star connection is shown below-

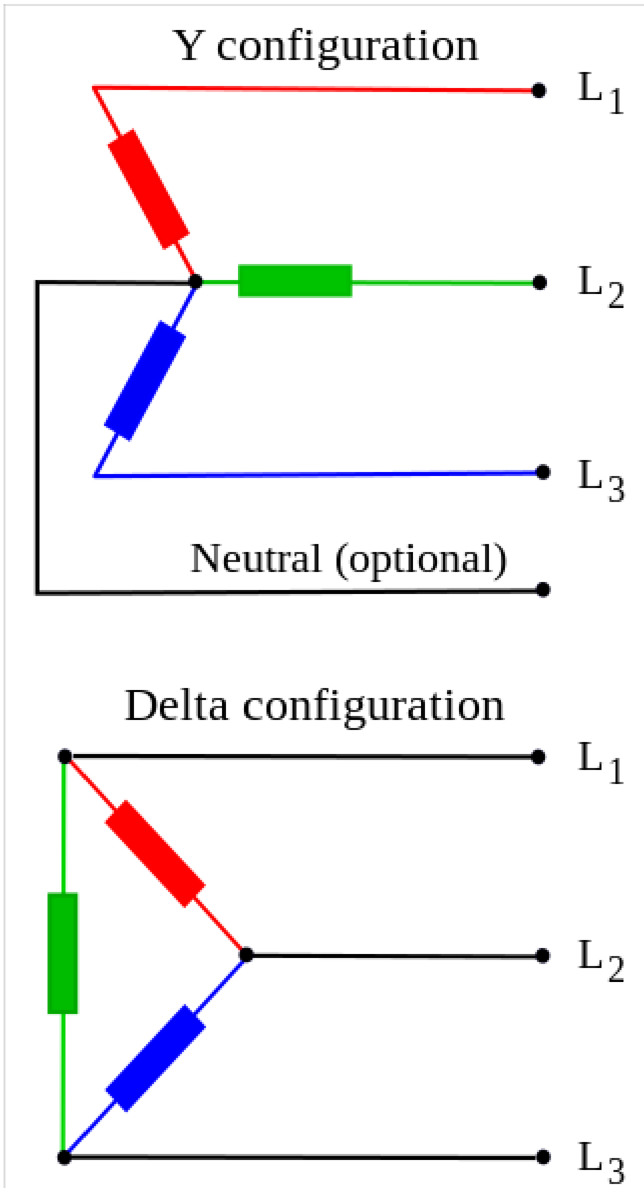


Fig. 2, 3

In star connection, the line voltage is √3 times of phase voltage. Line voltage is the voltage between two phases in three phase circuit and phase voltage is the voltage between one phase to the neutral line. And the current is same for both line and phase. It is shown as expression below

$$E_{line} = \sqrt{3}\, E_{phase} \text{ and } I_{Line} = I_{phase}$$

5.2.2 DELTA CONNECTION

In delta connection, there are three wires alone and no neutral terminal is taken. Normally delta connection is preferred for short distance due to the problem of unbalanced current in the circuit. The figure is shown below for delta connection. In the load station, ground can be used as the neutral path if required.

In delta connection, the line voltage is same with that of phase voltage. And the line current is √3 times of the phase current. It is shown as expressed below,

$$E_{line} = E_{phase} \text{ and } I_{Line} = \sqrt{3} I_{phase}$$

In a three phase circuit, star and delta connection can be arranged in four different ways-

1. Star-Star connection
2. Star-Delta connection
3. Delta-Star connection
4. Delta-Delta connection

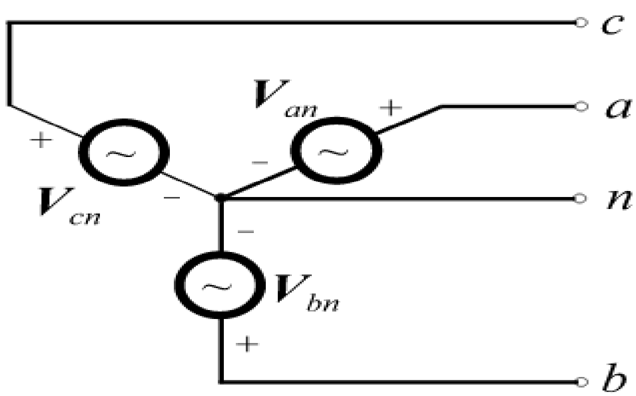
But the power is independent of the circuit arrangement of the three phase system. The net power in the circuit will be same in both star and delta connection. The power in three phase circuit can be calculated from the equation below,

$$P_{Total} = 3 \times E_{Phase} \times I_{Phase} \times PF$$

Since there are three phases, so the multiple of 3 is made in the normal power equation and the PF is the power factor. Power factor is a very important factor in three phase system and sometimes due to certain error, it is corrected by using capacitors.

5.3 ANALYSIS OF BALANCED THREE PHASE CIRCUITS(Lecture -18)

In a balanced system, each of the three instantaneous voltages has equal amplitudes, but is separated from the other voltages by a phase angle of 120. The three voltages (or phases) are typically labeled a, b and c. The common reference point for the three phase voltages is designated as the neutral connection and is labeled as n. The three sources V_{an}, V_{bn} and V_{cn} are designated as the line-to-neutral voltages in the three-phase system. An alternative way of defining the voltages in a balanced three-phase system is to define the voltage differences between the phases. These voltages are designated as line-to-line voltages. The line-to-line voltages can be expressed in terms of the line-to-neutral voltages by applying Kirchoff's voltage law to the generator circuit, which yields



$$V_{ab} = V_{an} - V_{bn}$$

$$V_{bc} = V_{bn} - V_{cn}$$

$$V_{ca} = V_{cn} - V_{an}$$

Fig.4

Either a positive phase sequence (abc) or a negative phase sequence(acb) as shown below.

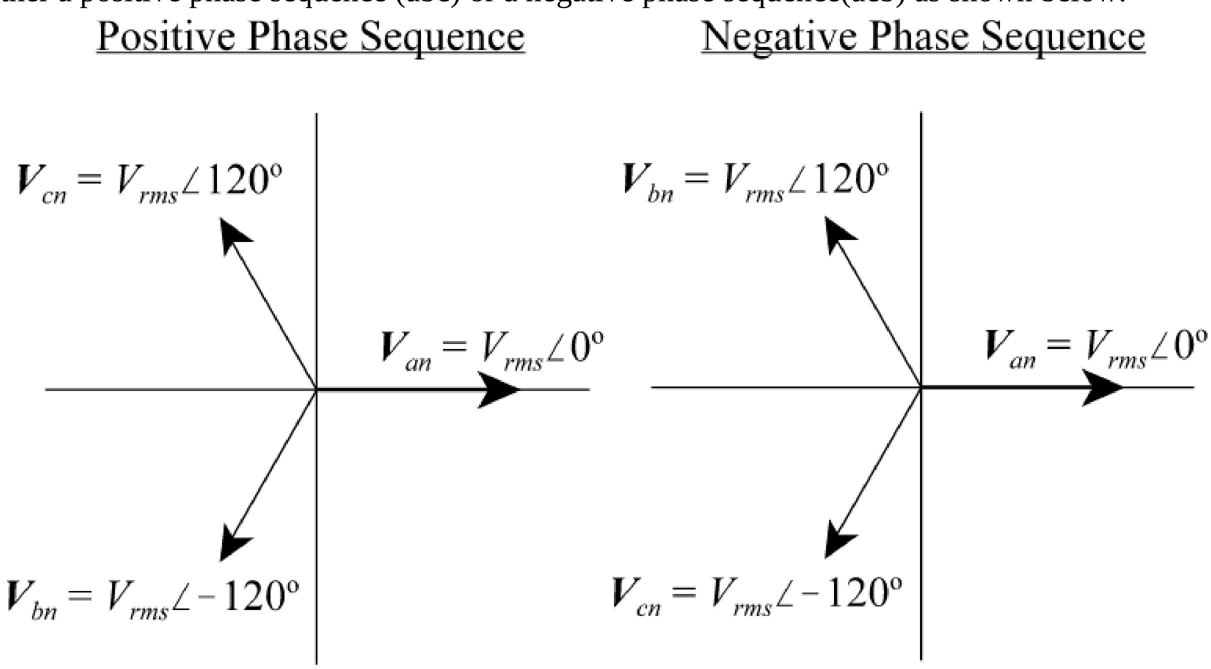


Fig.5

Inserting the line-to-neutral voltages for a positive phase sequence into the line-to-line equations yields

$$V_{ab} = V_{an} - V_{bn} = V_{rms}\angle 0^\circ - V_{rms}\angle{-120^\circ}$$

$$= V_{rms}\left[e^{j\,0^\circ} - e^{-j\,120^\circ}\right] = V_{rms}\{1 - [\cos(120^\circ) - jsin(120^\circ)]\}$$

$$= V_{rms}\left[1+\frac{1}{2}+j\frac{\sqrt{3}}{2}\right] = V_{rms}\left[\frac{3+j\sqrt{3}}{2}\right] = \sqrt{3}V_{rms}\left[\frac{\sqrt{3}+j1}{2}\right]$$

$$= \sqrt{3}V_{rms}\,[1\angle 30°]$$

$$= \sqrt{3}V_{rms}\,[\angle 30°]$$

$$V_{bc} = V_{bn} - V_{cn} = V_{rms}\angle{-120°} - V_{rms}\angle 120°$$

$$= V_{rms}\left[e^{-j120°} - e^{j120°}\right] = V_{rms}\{[\cos(120°) - jsin(120°)] - [cos(120°) + jsin(120°)]\}$$

$$= V_{rms}[-2jsin(120°)] = V_{rms}\left[-2j\frac{\sqrt{3}}{2}\right] = \sqrt{3}V_{rms}\,[1\angle -90°]$$

$$= \sqrt{3}V_{rms}\,[\angle{-90°}]$$

$$V_{ca} = V_{cn} - V_{an} = V_{rms}\angle 120° - V_{rms}\angle 0°$$

$$= V_{rms}\left[e^{j120°} - e^{j0°}\right] = V_{rms}\{[\cos(120°) + jsin(120°) - 1]\}$$

$$= V_{rms}\left[-\frac{1}{2}+j\frac{\sqrt{3}}{2}-1)\right] = V_{rms}\left[\frac{-3+j\sqrt{3}}{2}\right] = \sqrt{3}V_{rms}\left[\frac{-\sqrt{3}+j1}{2}\right]$$

$$= \sqrt{3}V_{rms}\,[1\angle 150°]$$

$$= \sqrt{3}V_{rms}\,[\angle 150°]$$

If we compare the line-to-neutral voltages with the line-to-line voltages, we find the following relationships,

Line-to-neutral voltages	**Line-to-line voltages**
$V_{an} = V_{rms}\angle 0°$	$V_{ab} = \sqrt{3}V_{rms}\angle 30°$
$V_{bn} = V_{rms}\angle -120°$	$V_{bc} = \sqrt{3}V_{rms}\angle -90°$
$V_{cn} = V_{rms}\angle 120°$	$V_{ca} = \sqrt{3}V_{rms}\angle 150°$

Line-to-line voltages in terms of line-to-neutral voltages:

$$V_{ab} = \sqrt{3}V_{an}e^{j30°}$$

$$V_{bc} = \sqrt{3}V_{bn}e^{j30°}$$

$$V_{ca} = \sqrt{3}V_{cn}e^{j\,30^{\circ}}$$

The equations above show that the magnitudes of the line-to-line voltages in a balanced three-phase system with a positive phase sequence are $\sqrt{3}$ times the corresponding line-to-neutral voltages and lead these voltages by 30°.

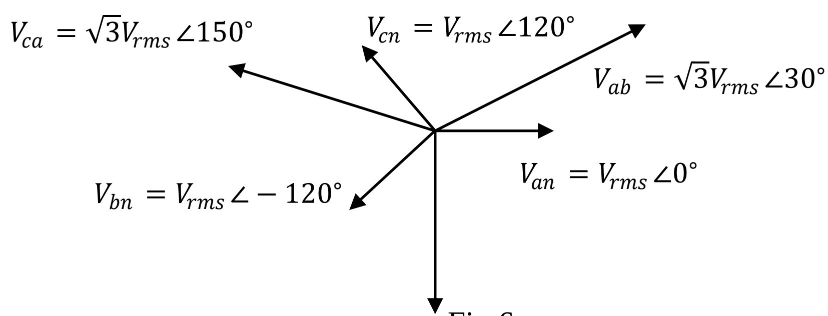


Fig.6

Balanced three-phase circuits consist of a balanced three-phase source and a balanced three-phase load can be of four varieties like Y-Y, Y-Δ, $\Delta - Y$ *or* $\Delta - \Delta$. Common analysis procedure for all the possible configurations are done.

5.3.1 EQUIVALENCE BETWEEEN A Y AND Δ CONNECTED SOURCE (Lecture -19)

Let $V_{RY} = V_L\angle 0^{\circ}$, $V_{YB} = V_L\angle{-120^{\circ}}$, and $V_{BR} = V_L\angle 120^{\circ}$ be the three line voltages observed in a three-phase circuit and let $I_R = I_L\angle -(\theta + 30^{\circ})$, $I_Y = I_L\angle -(\theta + 150^{\circ})$, and $I_B = I_L\angle -(\theta - 90^{\circ})$ be the observed line currents where the observed phase lag of first line current is expressed as 30° plus some angle designated by θ. If the source is within a black box and have to guess whether it is a Y-connected source or a Δ - connected source, it becomes difficult to resolve the using the observed line voltage phasors and line current phasors. This is so because both configurations shown in fig.7 will result in these observed line voltages and line currents.

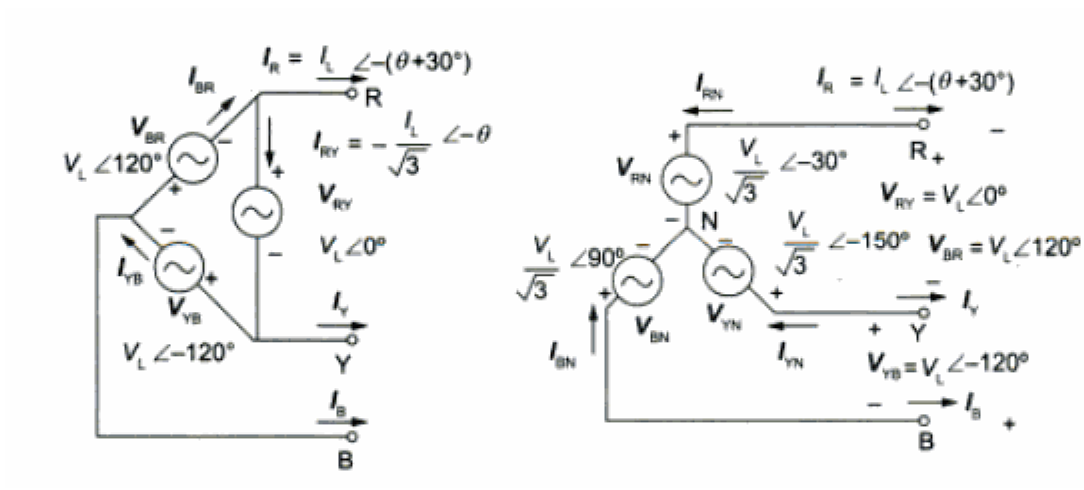


Fig.7

Therefore if a balanced three-phase source is actually Δ - connected, it may be replaced by an equivalent Y-connected source for purposes of analysis. The details of phase – source currents in the actual Δ - connected source can be obtained after the circuit problem is solved using the star equivalent source.

5.3.2 EQUIVALENCE BETWEEEN A Y AND Δ CONNECTED LOAD

The individual branch currents in the branches of delta in the actual Δ-connected load can be obtained after line quantities have been obtained by using its equivalent Y-connected model. Three-phase symmetry and $\frac{1}{\sqrt{3}}$ factor connecting line currents and phase currents in a Δ-connected system can be used for this purpose. The impedance to be used in Y-connected load is $\frac{1}{3}$ times the impedance present in Δ-connected load as shown in fig. 8.

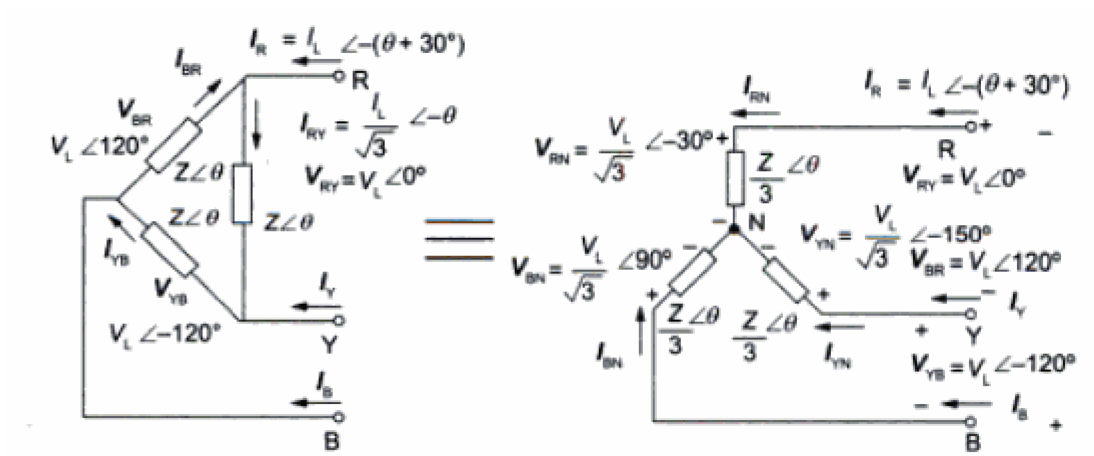


Fig.8

5.4 ANALYSIS OF UNBALANCED LOADS

Three-phase systems deliver power in enormous amounts to single-phase loads such as lamps, heaters, air-conditioners, and small motors. It is the responsibility of the power systems engineer to distribute these loads equally among the three-phases to maintain the demand for power fairly balanced at all times. While good balance can be achieved on large power systems, individual loads on smaller systems are generally unbalanced and must be analyzed as unbalanced three phase systems.

5.4.1 UNBALANCED DELTA CONNECTED LOAD

An unbalanced condition is due to unequal delta connected load.

Example 5.4.1.1 A delta connected load as shown in fig. 9 is connected across three-phase 100V supply. Determine all the line currents. Also draw the relevant phasor diagram showing all voltages and currents. Given $V_{AB} = 100\angle 0°$, $V_{BC} = 100\angle -120°$, $V_{CA} = 100\angle -240°$

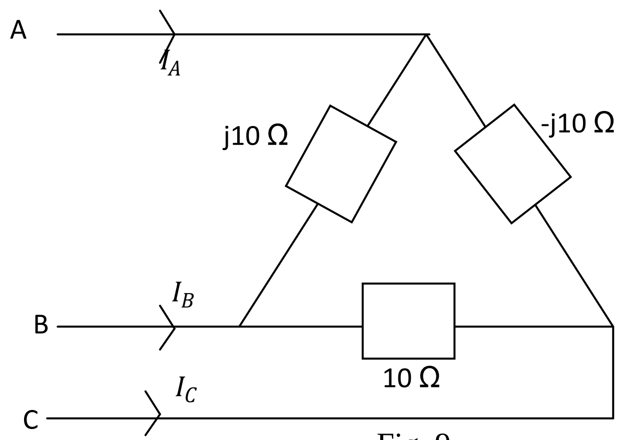


Fig. 9

Solution: Given: $V_{AB} = 100\angle 0°$, $V_{BC} = 100\angle -120°$, $V_{CA} = 100\angle -240°$

$$I_{AB} = \frac{V_{AB}}{Z_{AB}} = \frac{100\angle 0°}{j10} = 10\angle 0°\ A,$$

$$I_{BC} = \frac{V_{BC}}{Z_{BC}} = \frac{100\angle -120°}{10} = 10\angle -120°\ A,$$

$$I_{CA} = \frac{V_{CA}}{Z_{CA}} = \frac{100\angle -240°}{-j10} = 10\angle -150° A$$

$$I_A = I_{AB} - I_{CA} = 10\angle -90° - 10\angle -150° = 10\angle -30°\ A$$

$$I_B = I_{BC} - I_{AB} = 10\angle -120° - 10\angle -90° = 5.17\angle 165°\ A$$

$$I_C = I_{CA} - I_{BC} = 10\angle -150° - 10\angle -120° = 5.17\angle 135°\ A$$

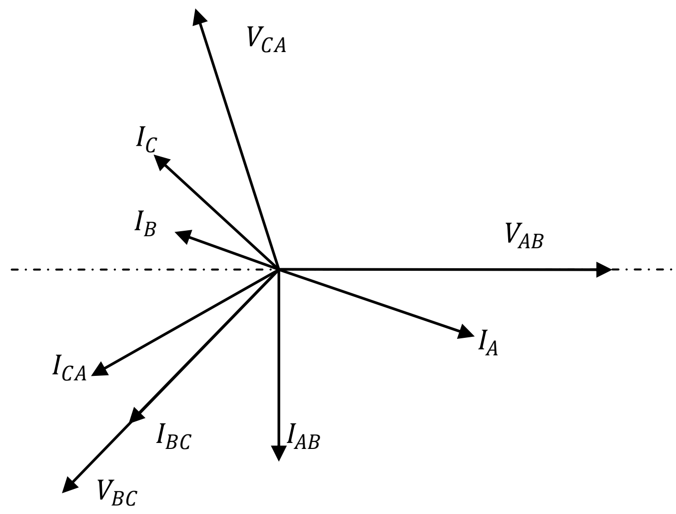


Fig.9 phasor diagram

5.4 NEUTRAL SHIFT

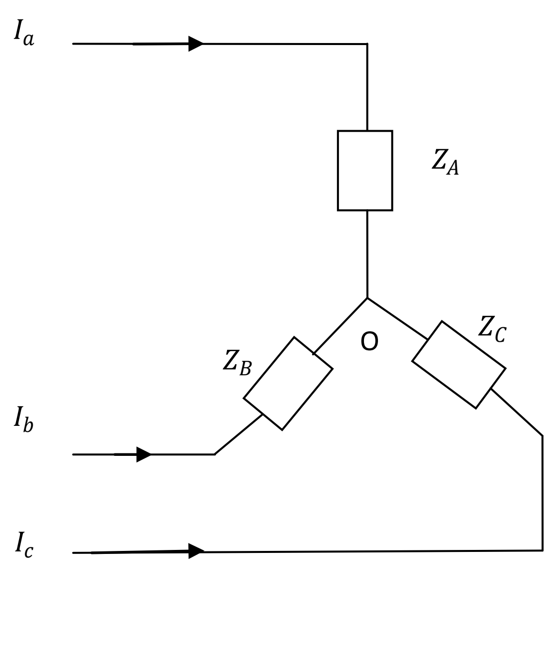


Fig. 10

For balanced three-phase, $I_N = 0$. For unbalanced system, I_N has some finite value, due to flow of unequal currents in each of the phases, voltage appears at the neutral and its value can be found as follows:

$$I_a + I_b + I_c = V_{a_0} Y_a + V_{b_0} Y_b + V_{c_0} Y_c$$

$$=(V_{an} - V_{on})Y_a + (V_{bn} - V_{on})Y_b + (V_{cn} - V_{on})Y_c$$

$$=V_{an} Y_a + V_{bn} Y_b + V_{cn} Y_c - V_{on}(Y_a + Y_b + Y_c)$$

Applying KCL at O, $I_a + I_b + I_c = 0$

$$V_{an} Y_a + V_{bn} Y_b + V_{cn} Y_c - V_{on}(Y_a + Y_b + Y_c) = 0$$

$$V_{on} = \frac{V_{an} Y_a + V_{bn} Y_b + V_{cn} Y_c}{Y_a + Y_b + Y_c} \dots\dots\dots\dots\dots\dots\dots\dots..(5.4.1)$$

The equation 5.4.1 gives the neutral shift.

5.5 SYMMETRICAL COMPONENTS (Lecture -20)

When an unbalanced three-phase fault occurs, we can solve the three-phase circuit using circuit theory. This is much more numerically complicated than the single phase circuit normally used in balanced three phase circuits. The degree of difficulty increases with the third power of the system size. For this reason, it is apparent that if we were to solve three different single-phase circuits, it would be numerically simpler than solving the one three-phase circuit in one set of equations. The purpose of this chapter is to break up the large three-phase circuit into three circuits, each one third the size of the whole system. Next, we solve the three components individually, and then combine the results to obtain the total system response.

5.5.1 FUNDAMENTALS OF SYMMETRICAL COMPONENTS

It was Fortescue in 1918 who developed the idea of breaking up asymmetrical three-phase voltages and currents into three sets of symmetrical components. These three basic components are:

(1) The positive sequence currents and voltages (known also as the "abc" and often denoted by the superscript "1" or " + ",) shown on the fig 9. There is a phase shift 120° between any two voltages.

$$|V_{a_1}| = |V_{b_1}| = |V_{c_1}|$$

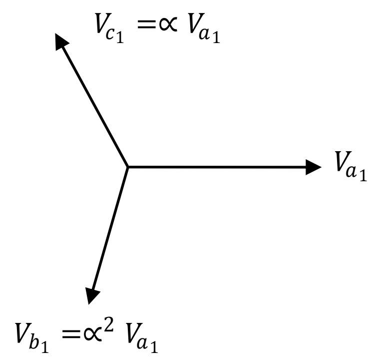


Fig. 9

(2) The negative sequence currents and voltages (known also as the "acb" and often denoted by the superscript "2" or " – ") . Note the sequence of the phasors is the opposite direction from the positive sequence (acb instead of abc). There is a phase shift 120° between any two voltages.

$$|V_{a_2}| = |V_{b_2}| = |V_{c_2}|$$

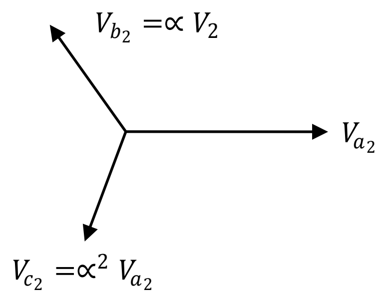


Fig. 10

(3) The zero sequence components of currents and voltages (often denoted by the superscript "0") .Note that these zero sequence phasors are all in-phase and equal in magnitude.

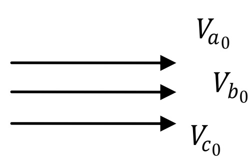


Fig. 11

Here the phase shift operator is $\propto$. $\propto$ the vector by 120°without changing magnitude.We will need the vector $\propto = 1\angle 120^\circ$, which is a unit vector at an angle of 120 degrees. It is easy to see that $\propto^2 = 1\angle 240^\circ = 1\angle -120^\circ$ and $\propto^3 = 1\angle 360^\circ = 1\angle 0^\circ$.
It is also clear that $1 + \propto + \propto^2 = 0$.

$$V_a = V_{a_\circ} + V_{a_1} + V_{a_2}$$

$$V_b = V_{b_\circ} + V_{b_1} + V_{b_2}$$

$$V_b = V_{a_\circ} + \propto^2 V_{a_1} + \propto V_{a_2}$$

$$V_c = V_{c_\circ} + V_{c_1} + V_{c_2}$$

$$V_c = V_{c_\circ} + \propto V_{a_1} + \propto^2 V_{a_2}$$

In matrix form,

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \propto^2 & \propto \\ 1 & \propto & \propto^2 \end{bmatrix} \begin{bmatrix} V_{a_0} \\ V_{a_1} \\ V_{a_2} \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.5.1.1)$$

In one line we can write the above equation 5.5.1.1.

$$V^{abc} = [A][V^{012}]$$

$$V^{012} = [A^{-1}][V^{abc}]$$

$$\begin{bmatrix} V_{a_0} \\ V_{a_1} \\ V_{a_2} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \propto^2 & \propto \\ 1 & \propto & \propto^2 \end{bmatrix}^{-1} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$\begin{bmatrix} V_{a_0} \\ V_{a_1} \\ V_{a_2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \propto & \propto^2 \\ 1 & \propto^2 & \propto \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$V_{a_0} = \frac{1}{3}(V_a + V_b + V_c)$$

$$V_{a_1} = \frac{1}{3}(V_a + \propto V_b + \propto^2 V_c)$$

$$V_{a_2} = \frac{1}{3}(V_a + \propto^2 V_b + \propto V_c)$$

Similarly, for current

$$I_{a_0} = \frac{1}{3}(I_a + I_b + I_c)$$

$$I_{a_1} = \frac{1}{3}(I_a + \propto I_b + \propto^2 I_c)$$

$$I_{a_2} = \frac{1}{3}(I_a + \propto^2 I_b + \propto I_c)$$

5.5.2 POWER IN TERMS OF SYMMETRICAL COMPONENTS

Suppose in abc sequence, 3Φ complex power, $S^{abc} = V_a I_a^* + V_b I_b^* + V_c I_c^*$

$$= [V_a + V_b + V_c]\begin{bmatrix} I_a^* \\ I_b^* \\ I_c^* \end{bmatrix}$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = V^{abc}, \quad \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = I^{abc}$$

$$S^{abc} = [V^{abc}]^T\left[I^{abc\,*}\right] \text{……………………..(5.5.2.1)}$$

5.2.2.1 RELATION BETWEEN ACTUAL QUANTITIES (a,b,c) AND SEQUENCE QUANTITIES

$$[V^{abc}] = [A][V^{012}]$$

$$[V^{abc}]^T = [A]^T[V^{abc}]^T$$

$$[I^{abc}] = [A][I^{012}]$$

From equation 5.2.2.1,

$$S^{abc} = [V^{012}]^T[A]^T[A]^*\,[I^{012}]^*$$

$$[A]^T.[A]^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \propto^2 & \propto \\ 1 & \propto & \propto^2 \end{bmatrix}^T \begin{bmatrix} 1 & 1 & 1 \\ 1 & \propto^2 & \propto \\ 1 & \propto & \propto^2 \end{bmatrix}^* = 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 3U$$

$$S^{abc} = [V^{012}]^T 3\,U\,[I^{012}]^* = 3[V^{012}]^T\,[I^{012}]^*$$

$$= 3[V_{a_0} \quad V_{a_0} \quad V_{a_0}]\begin{bmatrix} I_{a_0}{}^* \\ V_{a_0}{}^* \\ V_{a_0}{}^* \end{bmatrix}$$

$$= 3[V_{a_0}I_{a_0}{}^* + \quad V_{a_1}I_{a_1}{}^* + \quad V_{a_2}I_{a_2}{}^*]$$

$$S^{abc} = 3.S^{012}$$

************END OF MODULE -II ********************

UNIT V

FILTERS AND FOURIER ANALYSIS OF AC CIRCUITS

PASSIVE FILTERS

Frequency-selective or filter circuits pass to the output only those input signals that are in a desired range of frequencies (called pass band). The amplitude of signals outside this range of frequencies (called stop band) is reduced (ideally reduced to zero). Typically in these circuits, the input and output currents are kept to a small value and as such, the current transfer function is not an important parameter. The main parameter is the voltage transfer function in the frequency domain, Hv (jω) = Vo/Vi . As Hv (jω) is complex number, it has both a magnitude and a phase, filters in general introduce a phase difference between input and output signals. To minimize the number of subscripts, hereafter, we will drop subscript v of Hv. Furthermore, we concentrate on the the "open-loop" transfer functions, Hvo, and denote this simply by H(jω).

Low-Pass Filters

An ideal low-pass filter's transfer function is shown. The frequency between the pass- and-stop bands is called the cut-off frequency (ωc). All of the signals with frequencies below ωc are transmitted and all other signals are stopped.

In practical filters, pass and stop bands are not clearly defined, |H(jω)| varies continuously from its maximum toward zero. The cut-off frequency is, therefore, defined as the frequency at which |H(jω)| is reduced to 1/√2 = 0.7 of its maximum value. This corresponds to signal power being reduced by 1/2 as P ∝ V 2.

Band-pass filters

A band pass filter allows signals with a range of frequencies (pass band) to pass through and attenuates signals with frequencies outside this range.

Band

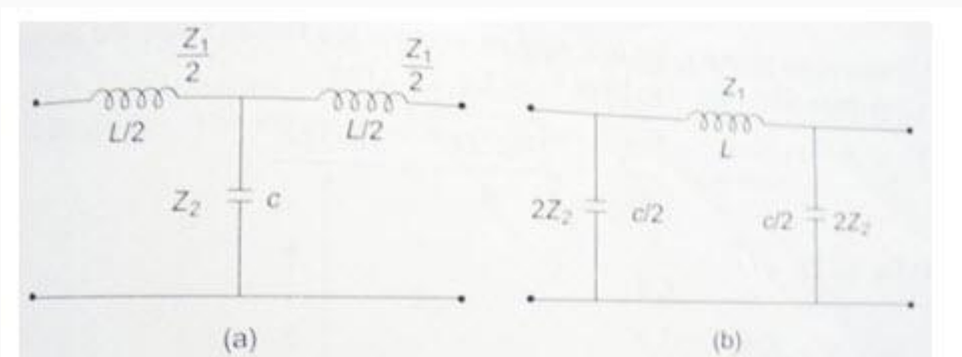
Constant – K Low Pass Filter

A network, either *T* or \[\pi\], is said to be of the constant-*k* type if Z_1 and Z_2 of the network satisfy the relation

$$Z_1Z_2 = k^2 \backslash$$

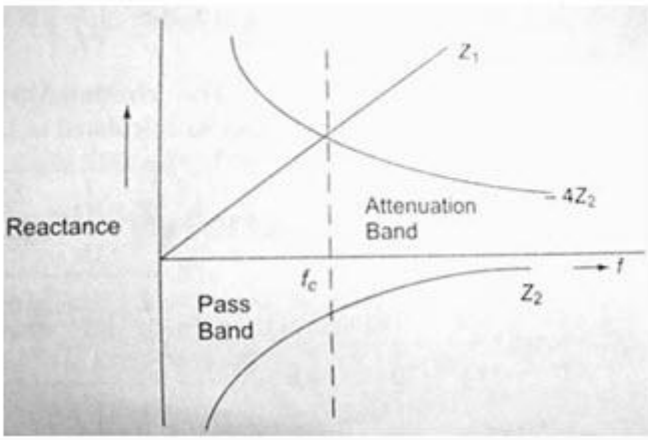
where Z_1 and Z_2 are impedance in the T and \[\pi\] sections as shown in Fig.17.8. Equation 17.20 states that Z_1 and Z_2 are inverse if their product is a constant, independent of frequency. *k* is a real constant, that is the resistance. *k* is often termed as design impedance or nominal impedance of the constant *k*-filter.

The constant *k*, *T* or \[\pi\] type filter is also known as the prototype because other more complex networks can be derived from it.



where $Z_1 = j\omega_L$ and $Z_2 = 1/j\omega_C$. Hence $Z_1Z_2 = {L \over C}={k^2}$ which is independent of frequency

The pass band can be determined graphically. The reactances of Z_1 and $4Z_2$ will vary with frequency as drawn in Fig.30.2. The cut-off frequency at the intersection of the curves Z_1 and $4Z_2$ is indicated as f_c. On the X-axis as $Z_1 = -4Z_2$ at cut-off frequency, the pass band lies between the frequencies at which $Z_1 = 0$, and $Z_1=-4Z_2$.



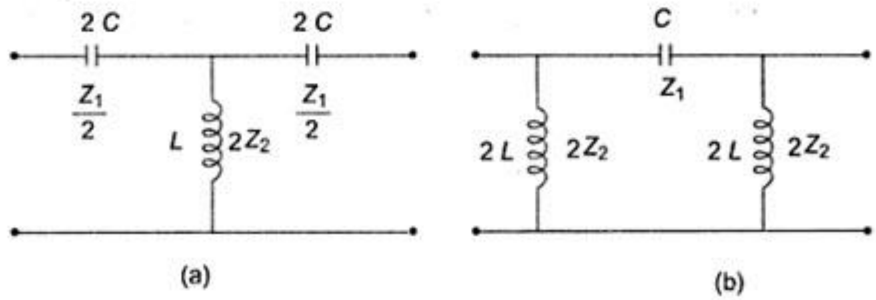
All the frequencies above f_c lie in a stop or attenuation band

The characteristic impedance of a π-network is given by

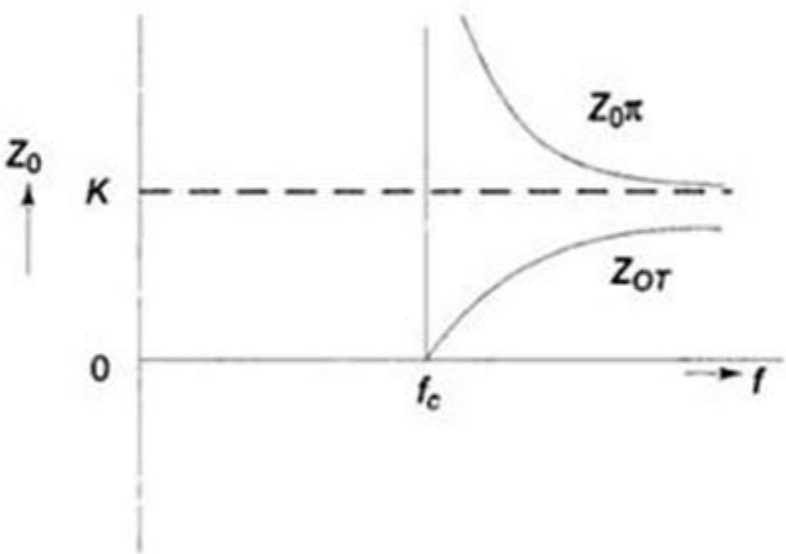
$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}} = \frac{k}{\sqrt{1-\left(\frac{f}{f_c}\right)^2}} \qquad (30.5)$$

Constant K-High Pass Filter

Constant K-high pass filter can be obtained by changing the positions of series and shunt arms of the networks shown in Fig.30.1. The prototype high pass filters are shown in Fig.30.5, where $Z_1 = -j/\omega_C$ and $Z_2 = j\omega L$.

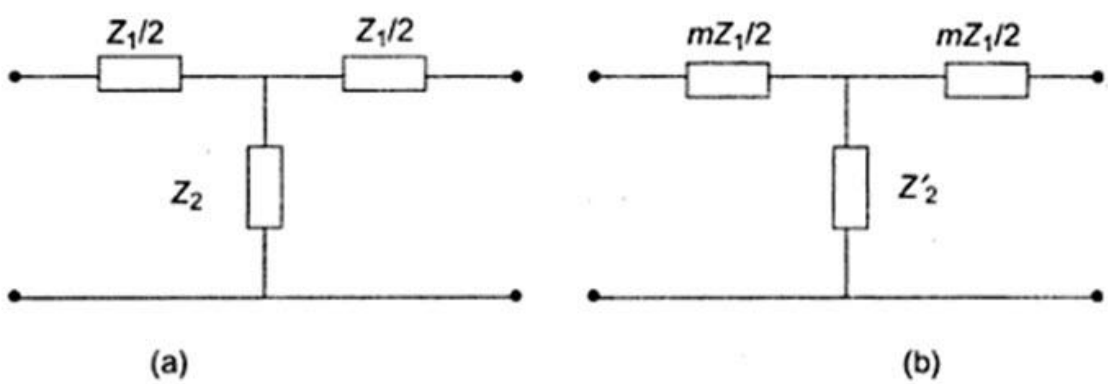


Again, it can be observed that the product of Z_1 and Z_2 is independent of frequency, and the filter design obtained will be of the constant k type.
The plot of characteristic impedance with respect to frequency is shown



m-Derived T-Section

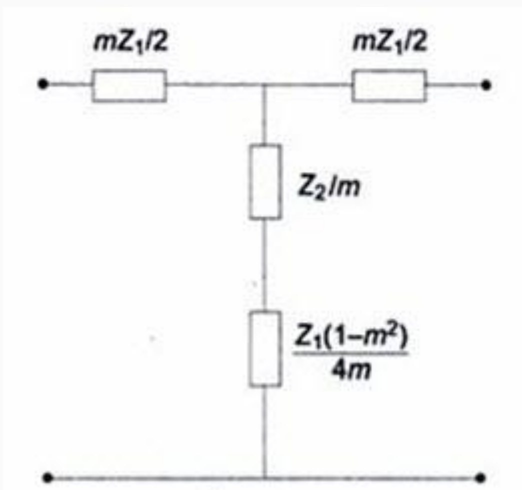
It is clear from previous chapter Figs 30.3 & 30.7 that the attenuation is not sharp in the stop band for k-type filters. The characteristic impedance, Z_0 is a function of frequency and varies widely in the transmission band. Attenuation can be increased in the stop band by using ladder section, i.e. by connecting two or more identical sections. In order to join the filter sections, it would be necessary that their characteristic impedance be equal to each other at all frequencies. If their characteristic impedances match at all frequencies, they would also have the same pass band. However, cascading is not a proper solution from a practical point of view. This is because practical elements have a certain resistance, which gives rise to attenuation in the pass band also. Therefore, any attempt to increase attenuation in stop band by cascading also results in an increase of 'a' in the pass band. If the constant k section is regarded as the prototype, it is possible to design a filter to have rapid attenuation in the stop band, and the same characteristic impedance as the prototype at all frequencies. Such a filter is called m-derived filter. Suppose a prototype T-network shown in Fig.31.1 (a) has the series arm modified as shown in Fig.31.1 (b), where m is a constant. Equating the characteristic impedance of the networks in we have



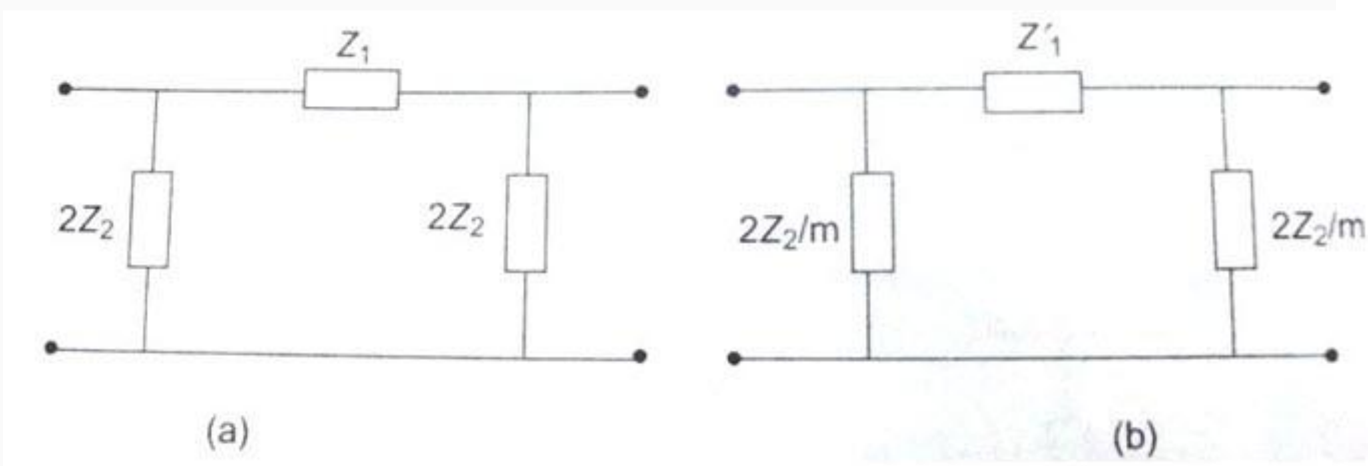
$Z_{0T} = Z_{0T'}$

where $Z_{0T'}$ is the characteristic impedance of the modified (m-derived) T-network.

Thus m-derived section can be obtained from the prototype by modifying its series and shunt arms. The same technique can be applied to \[\pi\] section network. Suppose a prototype p-network shown in Fig.31.3 (a) has the shunt arm modified as shown in Fig.31.3 (b).



The characteristic impedances of the prototype and its modified sections have to be equal for matching.



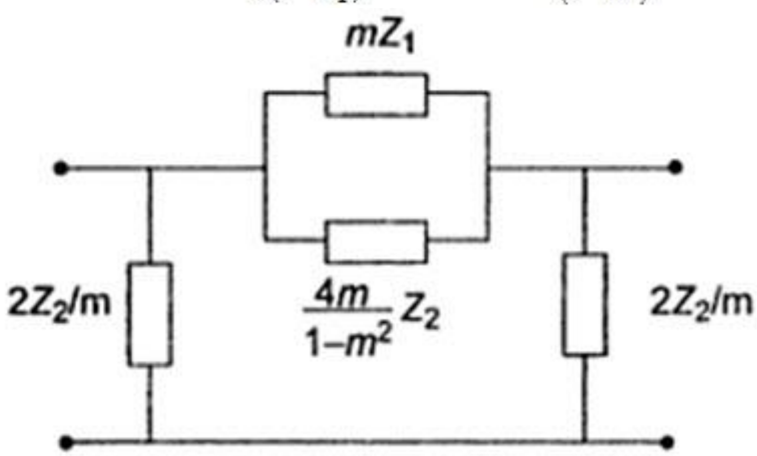
the characteristic impedance of the modified (m-derived) \[\pi\]-network

$$\therefore \qquad \sqrt{\frac{Z_1 Z_2}{1+\frac{Z_1}{4Z_2}}} = \sqrt{\frac{Z_1^{'}\frac{Z_2}{m}}{1+\frac{Z_1^{'}}{4.Z_2/m}}}$$

$$\text{Or} \qquad Z_1^{'} = \frac{Z_1 Z_2}{\frac{Z_1}{4m}+\frac{Z_2}{m}-\frac{mZ_1}{4}}$$

$$= \frac{Z_1 Z_2}{\frac{Z_2}{m}+\frac{Z_1}{4m}\left(1-m^2\right)}$$

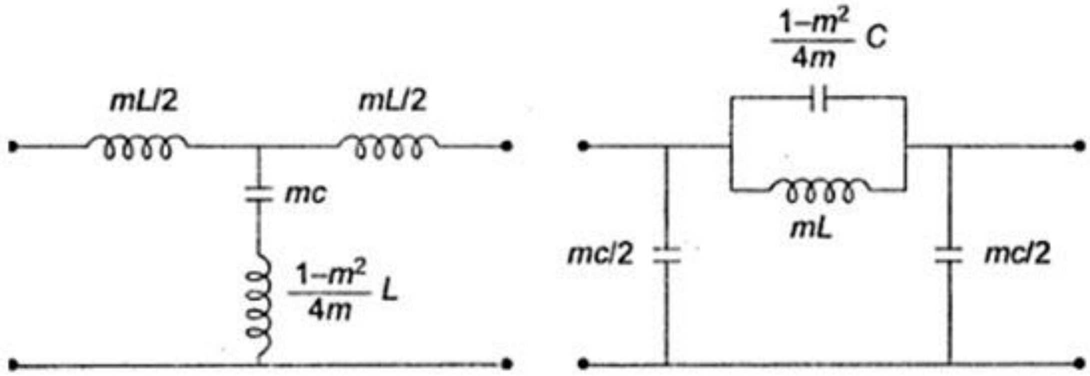
$$Z_1^{'} = \frac{Z_1 Z_2 \frac{4m^2}{\left(1-m^2\right)}}{\frac{Z_2 4m_2}{m\left(1-m_2\right)}+Z_1 m} = \frac{mZ_1 \frac{Z_2 4m}{\left(1-m^2\right)}}{mZ_1+\frac{Z_2 4m}{\left(1-m^2\right)}} \quad \text{...}(31.2)$$



the series arm of the m-derived \[\pi\] section is a parallel combination of mZ_1 and $4mZ_2/1\text{-}m^2$

m-Derived Low Pass Filter

In Fig.31.5, both m-derived low pass T and \[\pi\] filter sections are shown. For the T-section shown Fig.31.5 (a), the shunt arm is to be chosen so that it is resonant at some frequency f_x above cut-off frequency f_c its impedance will be minimum or zero. Therefore, the output is zero and will correspond to infinite attenuation at this particular frequency.



(a) (b)

$$m\omega_r L = \frac{1}{\left(\frac{1-m^2}{4M}\right)\omega_r C}$$

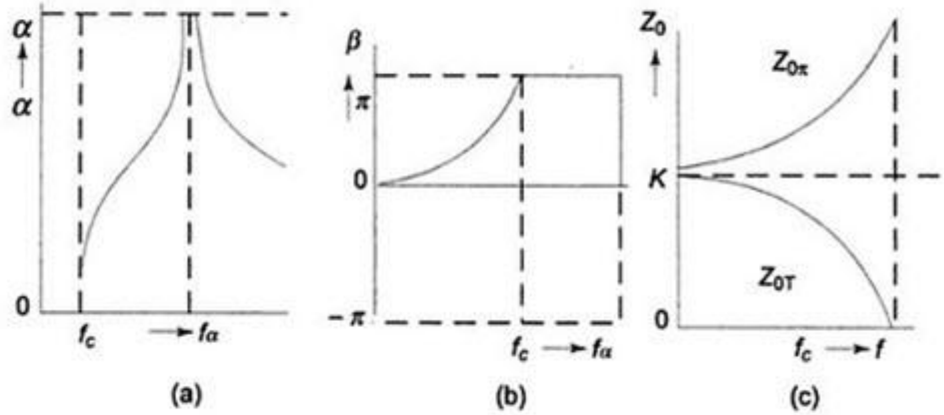
$$\omega_r^2 = \frac{4}{LC\left(1-m^2\right)}$$

$$f_r = \frac{1}{\pi\sqrt{LC\left(1-m^2\right)}}$$

The variation of attenuation for a low pass m-derived section can be verified

$$\therefore \qquad \alpha = 2\cosh^{-1}\frac{m\frac{f}{f_c}}{\sqrt{1-\left(\frac{f}{f_\alpha}\right)^2}}$$

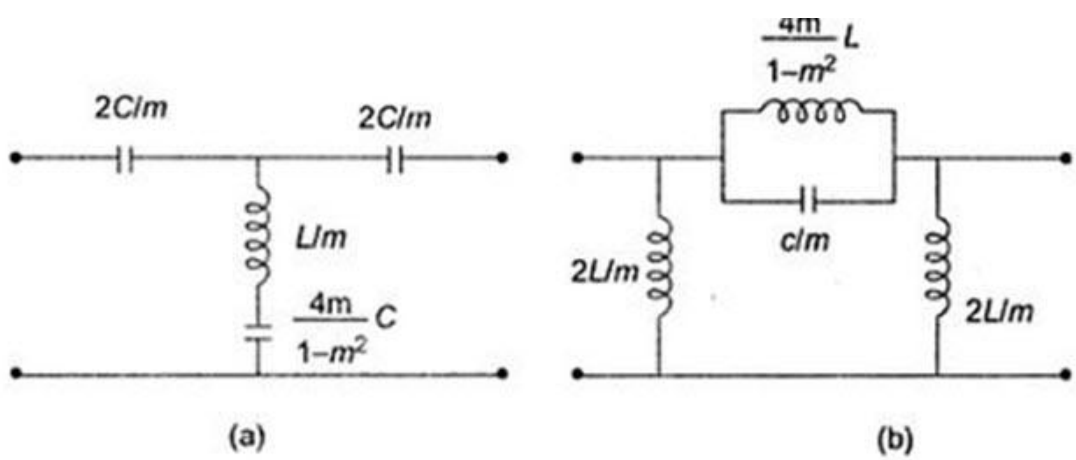
And $$\beta = 2\sin^{-1}\sqrt{\left|\frac{Z_1}{4Z_1}\right|} = 2\sin^{-1}\frac{m\frac{f}{f_c}}{\sqrt{1-\left(\frac{f}{f_c}\right)^2\left(1=m\right)^2}}$$



M-derived High Pass Filter

If the shunt arm in T-section is series resonant, it offers minimum or zero impedance. Therefore, the output is zero and, thus, at resonance frequency, or the frequency corresponds to infinite attenuation.

$$\omega_r \frac{L}{m} = \frac{1}{\omega_r \frac{4m}{1-m^2} C}$$

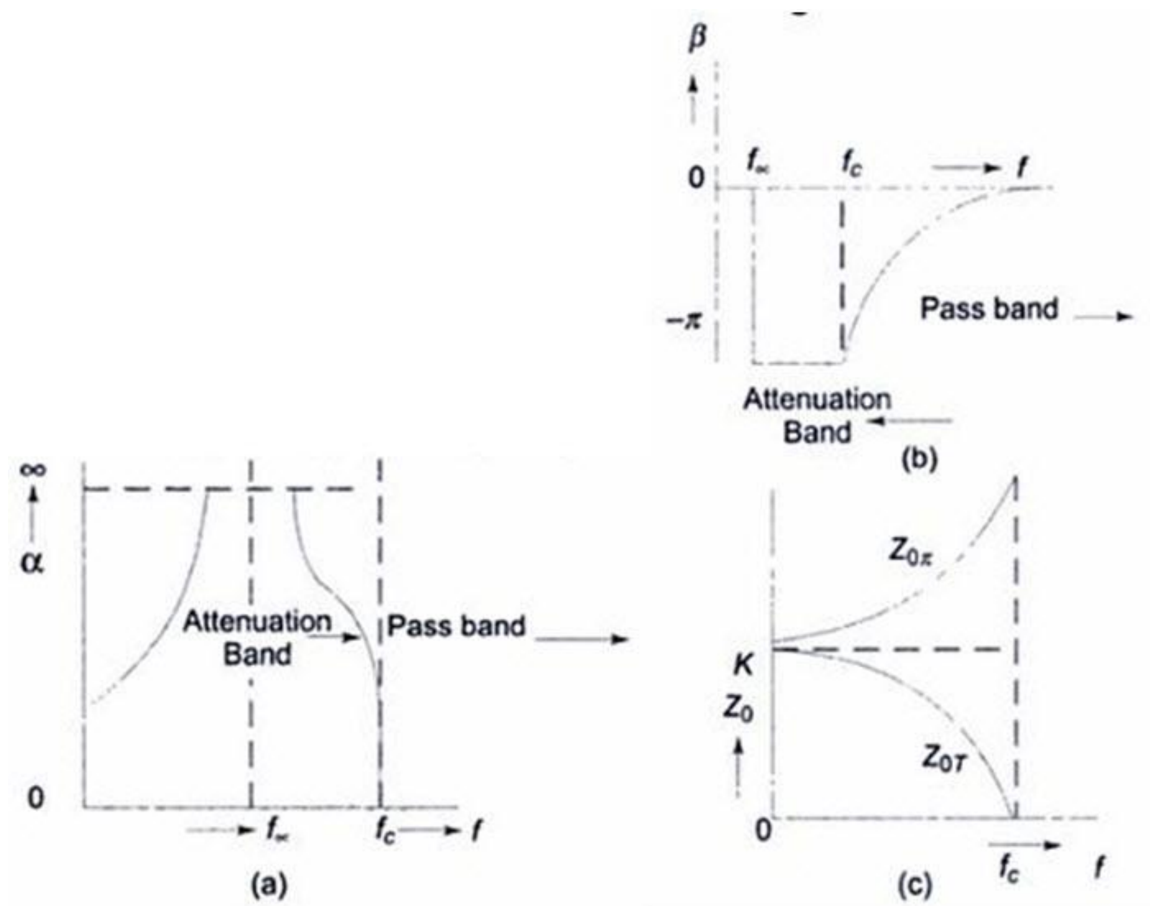


the m-derived \[\pi\]-section, the resonant circuit is constituted by the series arm inductance and capacitance.

$$\frac{4m}{1-m^2}\omega_r L = \frac{1}{\frac{\omega_r}{m}C}$$

$$\omega_r^2 = \omega_\alpha^2 = \frac{1-m^2}{4LC}$$

$$\omega_\alpha = \frac{\sqrt{1-m^2}}{2\sqrt{LC}} \; or \; f_\alpha = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}}$$



Fourier Series and Fourier Integrals

The type of series that can represent a a much larger class of functions is called Fourier Series. These series have the form

$$g(t) = a_0 + \sum_{m=1}^{\infty} a_m \cos\left(\frac{2\pi mt}{T}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi nt}{T}\right)$$

$$= \sum_{m=0}^{\infty} a_m \cos\left(\frac{2\pi mt}{T}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi nt}{T}\right)$$

$$a_0 = \frac{1}{T}\int_0^T f(t)dt$$

$$a_m = \frac{2}{T}\int_0^T f(t)\cos\left(\frac{2\pi mt}{T}\right)dt$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin\left(\frac{2\pi nt}{T}\right)dt$$

The coefficients an and bn are called Fourier coefficients. this series represents a periodic function with period T. To represent function f(x) in this way, the function has to be (1) periodic with just a finite number of maxima and minima within one period and just a finite number of discontinuities, (2) the integral over one period of |f(x)| must converge. If these conditions are satisfied and one period of f(x) is given on an interval (x0, x0 + T), the Fourier coefficients an and bn can be computed using the above formulae.

Fourier Transformation

The Fourier transform is an integral operator meaning that it is defined via an integral and that it maps one function to the other. If you took a differential equations course, you may recall that the Laplace transform is another integral operator you may have encountered. The Fourier transform represents a generalization of the Fourier series. Recall that the Fourier series is f(t) = P∞ n=−∞ cne 2nπit T . The sequence cn can be regarded as a function of n and is called Fourier spectrum of f(t). We can think of c(n) being another representation of f(t), meaning that f(t) and c(n) are different representations of the same object. Indeed: given f(t) the coefficients c(n) can be computed and, conversely, given c(n), the Fourier series with coefficients c(n) defines a function f(t). We can plot c(n) as a function of n (and get a set of infinitely many equally spaced points). In this case we think of c as a function of n, the wave number. We can also think of c as a function of ω = 2nπ T , the frequency. Note that if T is large, then ω is small and the function T cn becomes a continuous function of ω for T → ∞. Note also that if we let T → ∞, the requirement that f(t) is periodic can be waved since the period becomes (−∞,∞). The Fourier Transform F(ω) of f(t) is the limit of the continuous function √ 1 2π T cn when T → ∞.

Linearity
The F.T. is linear:

$$\mathcal{F}[af(t)+bg(t)] = a\mathcal{F}[f(t)] + b\mathcal{F}[g(t)]$$

Time/Frequency Duality
The duality property is one that is not shared by the Laplace transform. While slightly confusing perhaps at first, it essentially doubles the size of our F.T. table. The duality property follows from the similarity of the forward and inverse F.T. It states that if

$$f(t) \Leftrightarrow F(\omega)$$

then

$$F(t) \Leftrightarrow 2\pi f(-\omega)$$

where the function on the left is the function of time and the function on the right is the function of frequency.

Scaling Property

We have seen this for Laplace transforms: If

$$f(t) \Leftrightarrow F(\omega)$$

then

$$f(at) \Leftrightarrow \frac{1}{|a|}F(\omega/a)$$

Time-Shift Property
If

$$f(t) \Leftrightarrow F(\omega)$$

then

$$f(t-t_0) \Leftrightarrow F(\omega)e^{-j\omega t_0}$$

Frequency-shift Property
This innocuous-looking property forms a basis for every radio and TV transmitter in the world! It simply states that if

$$f(t) \Leftrightarrow F(\omega)$$

then

$$f(t)e^{j\omega_0 t} \Leftrightarrow F(\omega-\omega_0)$$

Convolution Property
If

$$f_1(t) \Leftrightarrow F_1(\omega) \quad \text{and} \quad f_2(t) \Leftrightarrow F_2(\omega)$$

then

$$f_1(t) * f_2(t) \Leftrightarrow F_1(\omega)F_2(\omega)$$

(where $*$ is convolution) and

$$f_1(t)f_2(t) \Leftrightarrow \frac{1}{2\pi}F_1(\omega) * F_2(\omega)$$

Time Differentiation

If

$$f(t) \Leftrightarrow F(\omega)$$

then

$$\frac{df}{dt} \Leftrightarrow j\omega F(\omega)$$

Time Integration

We saw above that

$$\int_{-\infty}^{t} f(x)dx \Leftrightarrow \frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$$

If f is zero mean (i.e. $F(0) = 0$) then

$$\int_{-\infty}^{t} f(x)dx \Leftrightarrow \frac{F(\omega)}{j\omega}$$

REFERENCES:

1. Electric Circuits , A. Chakrabarthy, Dhanpatrai & Sons
2. Network Theory : A. Sudhakar Shyammohan Palli